① • $D = \mathbb{R} \setminus \{-1\}$

a) • $f(x) = 0$ für $x^2 - 4x + 4 = 0$

$(x-2)^2 = 0 \Rightarrow x = 2$

• $f'(x) = \dfrac{(2x-4)\cdot(x+1) - 1\cdot(x^2-4x+4)}{(x+1)^2} = 0$ | · Nenner

$2x^2 + 2x - 4x - 4 - x^2 + 4x - 4 = 0$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0 \Rightarrow x = \begin{cases} 2 \\ -4 \end{cases}$

$\underbrace{E_1(2|0)}_{\text{Tiefpkt}}$ $\underbrace{E_2(-4|-12)}_{\text{Hochpkt}}$

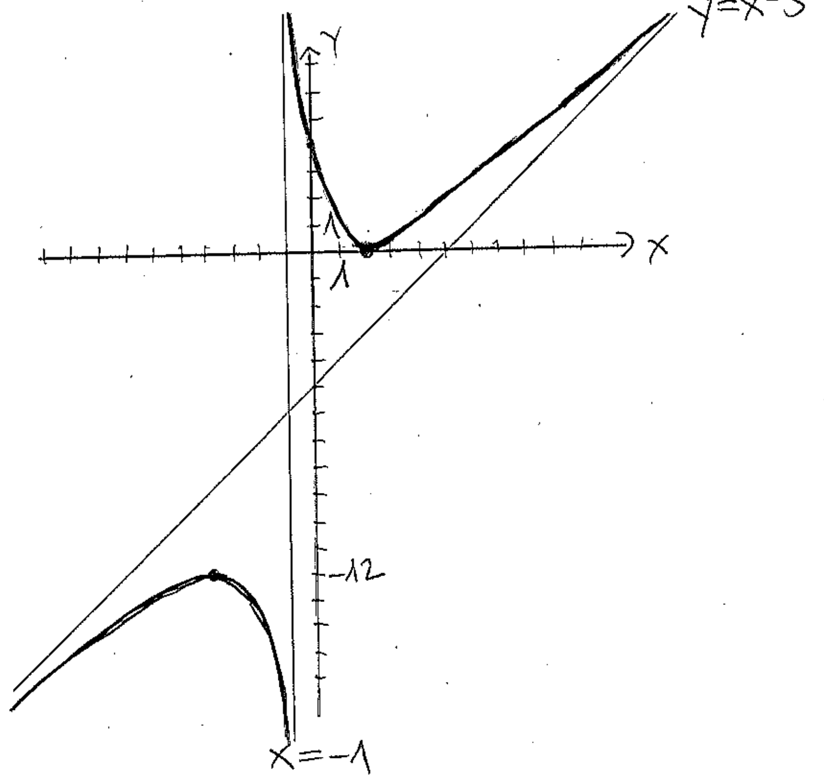
• Asymptoten: $x = -1$

$f(x) = x - 5 + \dfrac{9}{x+1} \xrightarrow{x \to \infty} 0$

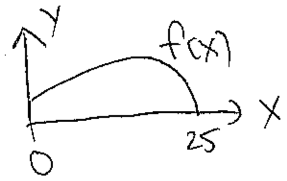
$\Rightarrow y = x - 5$

• $\lim\limits_{\substack{x \to -1 \\ x < -1}} f(x) = -\infty$ $\lim\limits_{\substack{x \to -1 \\ x > -1}} f(x) = +\infty$

• Wendepunkte bei $f''(x) = 0$
keine Lösung ⇒ keine Wendepunkte



b)



i) $f(0) = \frac{2}{6}\sqrt{25} = \frac{5}{3} \Rightarrow d = \frac{10}{3} = 3{,}3\,m$

ii) Länge Definitionsbereich → 25 m

iii) $f'(x) = 0$ für $x = 16\,m$

iv) $f(16) = 9 \Rightarrow$ Breite 18 m

v) $V_x = \pi \cdot \int_0^{25} [f(x)]^2\,dx = 3858{,}8\,m^3$

② a) $\hookrightarrow g \to E: \; 2(6+4t)+3(8+3t)+7(2+2t)+12=0$

$$12+8t+24+9t+14+14t+12=0$$

$$31t+62=0$$

$$t=-2$$

$\xrightarrow{t=-2} g: \; \underline{\underline{S(-2|2|-2)}}$

b) $$d(P,E)=\frac{2\cdot(6)+3(8)+7(2)+12}{\sqrt{2^2+3^2+7^2}}=\frac{62}{\sqrt{62}}$$

$$=\sqrt{62}\approx \underline{\underline{7{,}874}}$$

c) lot: $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}6\\8\\2\end{pmatrix}+t\begin{pmatrix}2\\3\\7\end{pmatrix}$

$\hookrightarrow l \to E \quad 2(6+2t)+3(8+3t)+7(2+7t)+12=0$

$$12+4t+24+9t+14+49t+12=0$$

$$62t+62=0$$

$$t=-1$$

Spiegelpunkt $t=-2 \Rightarrow \underline{\underline{P'(2|2|-12)}}$

d) $g': \begin{pmatrix}x\\y\\z\end{pmatrix}=\underset{\uparrow\; P'}{\begin{pmatrix}2\\2\\-12\end{pmatrix}}+t\underset{\overrightarrow{P'S}}{\begin{pmatrix}-4\\0\\10\end{pmatrix}}$

e) $$\sin(\alpha)=\frac{\begin{pmatrix}4\\3\\2\end{pmatrix}\cdot\begin{pmatrix}2\\3\\7\end{pmatrix}}{\sqrt{29}\sqrt{62}}=0{,}731$$

$$\alpha=\arcsin(0{,}731)\cong \underline{\underline{47^\circ}}$$

f) $$d=\frac{\left|\begin{pmatrix}6\\8\\2\end{pmatrix}\times\begin{pmatrix}4\\3\\2\end{pmatrix}\right|}{\left|\begin{pmatrix}4\\3\\2\end{pmatrix}\right|}=\frac{\left|\begin{pmatrix}10\\-4\\-14\end{pmatrix}\right|}{\sqrt{27}}=\frac{\sqrt{312}}{\sqrt{27}}$$

$$=\underline{\underline{3{,}400}}$$

g) $\vec{n}_F=\begin{pmatrix}-3\\2\\0\end{pmatrix}\times\begin{pmatrix}0{,}5\\2\\-1\end{pmatrix}=\begin{pmatrix}-2\\-3\\-7\end{pmatrix}$ $\parallel$ zu $\vec{n}_E=\begin{pmatrix}2\\3\\7\end{pmatrix}$

$\Rightarrow$ E und F sind entweder parallel oder identisch

$\rightarrow$ Liegt $\begin{pmatrix}2\\8\\-4\end{pmatrix}$ auf E ?

$\hookrightarrow \; 2(2)+3(8)+7(-4)+12 \overset{?}{=} 0$

$$4+24-28+12\neq 0$$

$\Rightarrow$ nicht auf E $\Rightarrow \underline{\underline{E \parallel F}}$

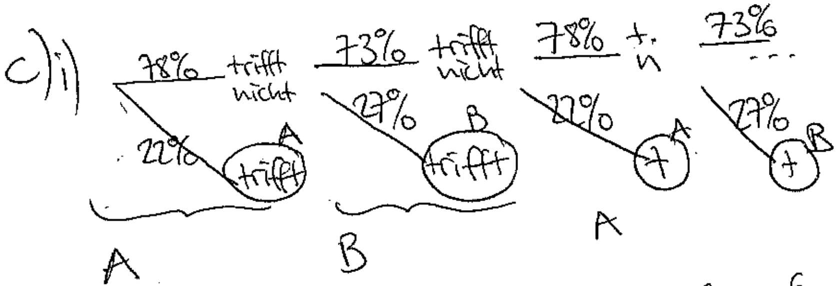
③ a) $78\% \cdot 73\% = \underline{\underline{59{,}94\%}}$

b) i) $B(12;\,0{,}22;\,2) = \underline{\underline{0{,}266}}$

ii) $B(12;\,0{,}22;\,0) = \underline{\underline{0{,}051}}$

iii) $B(12;\,0{,}22;\,k \geq 4) = \underline{\underline{0{,}261}}$

c) i)



ii) $\underbrace{78\% \cdot 73\% \cdot 78\% \cdot 73\% \cdot \ldots}_{12 \text{ Faktoren}} = 78\%^6 \cdot 73\%^6$

$= \underline{\underline{3{,}41\%}}$

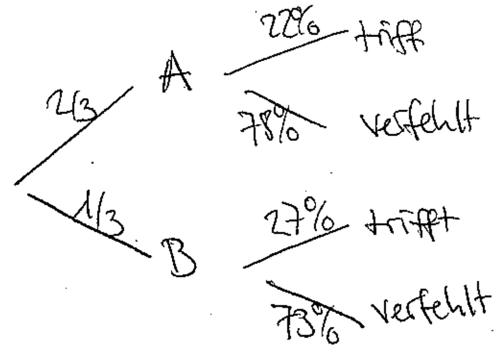
iii) $P(A \text{ siegt}) = 22\% + 78\% \cdot 73\% \cdot 22\% + (78\% \cdot 73\%)^2 \cdot 22\% + \ldots$

unendliche GF mit $a_1 = 22\%$
$q = 78\% \cdot 73\%$

$S_\infty = \dfrac{22\%}{1 - 78\% \cdot 73\%} = \underline{\underline{51{,}09\%}}$ ⇒ A leicht besser

(B hat $1 - 51{,}09\% = 48{,}91\%$)

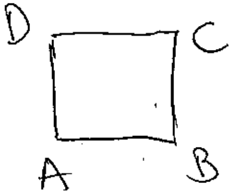
③ d)



$P(A \text{ schoss} \mid \text{Versuch verfehlt}) = \dfrac{\frac{2}{3} \cdot 78\%}{\frac{2}{3} \cdot 78\% + \frac{1}{3} \cdot 73\%}$

$= \underline{\underline{68{,}12\%}}$

④ a)



$$\vec{OD} = \vec{OC} + \vec{BA}$$
$$= \begin{pmatrix}12\\9\\0\end{pmatrix} + \begin{pmatrix}-21\\3\\0\end{pmatrix} = \begin{pmatrix}-9\\12\\0\end{pmatrix}$$

$\underline{\underline{D(-9|12|0)}}$

b) i) Seite $s = |\vec{BA}| = \sqrt{21^2+3^2} = \sqrt{450}$

$\Rightarrow \underline{\underline{G = 450\ m^2}}$

ii) Da überall auf ABC die z-Koordinate 0 ist, liegt die Pyramide auf der xy-Ebene $z=0$

$\Rightarrow$ Höhe ist 9m (weil $S(0|0|9)$)

$$\Rightarrow V = \frac{G \cdot h}{3} = \frac{450 \cdot 9}{3} = \underline{\underline{1350\, m^3}}$$

c) i) Horizontale $\vec{v}\,' = \begin{pmatrix}1\\2\\0\end{pmatrix}$

$$\cos(\alpha) = \frac{\begin{pmatrix}1\\2\\0\end{pmatrix} \cdot \begin{pmatrix}1\\2\\-2\end{pmatrix}}{\sqrt{5} \cdot \sqrt{9}} = \frac{5}{3\sqrt{5}} = 0{,}74\ldots$$

$\Rightarrow \alpha = 41{,}81°$

ii) Licht: $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}0\\0\\9\end{pmatrix} + t\begin{pmatrix}1\\2\\-2\end{pmatrix}$

Schneiden mit xy-Ebene $z=0$

$\Rightarrow 0 = 9 - 2t \Rightarrow t = 4{,}5$

$\underline{\underline{\bar{S} = (4{,}5|9|0)}}$

d) i) Ebene_{BCS}: $\vec{n} = \begin{pmatrix}3\\21\\0\end{pmatrix} \times \begin{pmatrix}-9\\12\\9\end{pmatrix} = \begin{pmatrix}189\\-27\\225\end{pmatrix} \longrightarrow$

$\vec{n} = \begin{pmatrix}21\\-3\\25\end{pmatrix}$

$21x - 3y + 25z + D = 0$ | setze S ein

$0 - 0 + 225 + D = 0$

$D = -225$

$$\boxed{E_{BCS}:\ 21x - 3y + 25z - 225 = 0}$$

ii) Mitte $\overline{AD}$: $M_{AD} = \frac{1}{2}\begin{pmatrix}-12-9\\-9+12\\0+0\end{pmatrix} = \begin{pmatrix}-10{,}5\\1{,}5\\0\end{pmatrix}$

$$d(M_{AD}, E_{BCC}) = \frac{21(-10{,}5) - 3(1{,}5) + 25 \cdot (0) - 225}{\sqrt{21^2+3^2+25^2}}$$

$$= \frac{-450}{\sqrt{1075}} = \underline{\underline{13{,}725}}$$

5) a)

$$\begin{array}{ll|l}
① f(0)=0 & \rightarrow & e=0 \\
② f(2)=2{,}4 & \rightarrow & a\cdot 2^4+b\cdot 2^3+c\cdot 2^2+d\cdot 2+e=2{,}4 \\
③ f'(2)=0 & \rightarrow & 4a\cdot 2^3+3b\cdot 2^2+2c\cdot 2+d=0 \\
④ f(4)=3{,}2 & \rightarrow & a\cdot 4^4+b\cdot 4^3+c\cdot 4^2+d\cdot 4+e=3{,}2 \\
⑤ f''(4)=0 & \rightarrow & 12a\cdot 4^2+6b\cdot 4+2c=0
\end{array}$$

$\Rightarrow \underline{\underline{f(x)=-0{,}05x^4+0{,}6x^3-2{,}4x^2+4x}}$

b) $\underline{\underline{f'(4)=0{,}8}}$

c) $y=0{,}8x+q \quad \Big| \begin{array}{l} x=4 \\ y=3{,}2 \end{array}$

$3{,}2=0{,}8\cdot 4+q$

$\Rightarrow q=0 \quad \Rightarrow \boxed{y=0{,}8x}$

d) $m=0{,}8$

$\Rightarrow \alpha=\arctan(0{,}8)=\underline{\underline{38{,}66°}}$

e) Nein, da z.B beim Start in O gilt: $f'(0)=4$
(und $f'(4)=0{,}8$ bei S)

f) $\overline{f(x)}_{[0,5]}=\frac{1}{5}\cdot\int_0^5 f(x)\,dx=\frac{1}{5}\cdot 12{,}5$

$=\underline{\underline{2{,}5 \text{ km ü } 0}}$

⑥ a) i) $\frac{1}{6}\cdot\frac{1}{7}\cdot\frac{1}{5}=\frac{1}{210}=\underline{\underline{0{,}00476}}$

ii) $\underbrace{\frac{1}{6}\cdot\frac{2}{7}\cdot 0}_{\text{Orange}} + \underbrace{\frac{2}{6}\cdot\frac{1}{7}\cdot\frac{1}{5}}_{\text{Apfel}} + \underbrace{\frac{1}{6}\cdot\frac{3}{7}\cdot\frac{2}{5}}_{\text{Banane}} + \underbrace{0}_{\text{Birne}}$

$\frac{8}{210} = \underline{\underline{0{,}0381}}$

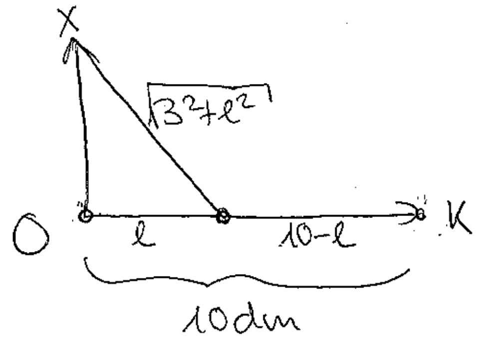
iii)

X	100	1	−2
	$\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}$ $=\frac{1}{216}$	$\frac{2}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}\cdot 6$ Anz ef. $=\frac{1}{6}$	Rest $1-\frac{1}{216}-\frac{1}{6}$ $=\frac{179}{216}$

$\Rightarrow E(X) = \frac{1}{216}\cdot 100 + \frac{1}{6}\cdot 1 + \frac{179}{216}\cdot(-2)$

$= \underline{\underline{-1{,}027}}$

⑥ b) Ziel: $4\cdot\sqrt{3^2+\ell^2}+2\cdot(10-\ell)$ minimal



ZF: $4\sqrt{9+\ell^2}+20-2\ell = 4(9+\ell^2)^{1/2}+20-2\ell$

ZF': $4\cdot\frac{1}{2}(9+\ell^2)^{-1/2}\cdot 2\ell-2 = 0$

$4\ell\cdot\frac{1}{\sqrt{9+\ell^2}} = 2 \quad |\cdot\sqrt{\ }$

$4\ell = 2\sqrt{9+\ell^2} \quad |(\ )^2$

$16\ell^2 = 4(9+\ell^2)$

$12\ell^2 = 36$

$\ell^2 = 3$

$\ell = \sqrt{3} \approx 1{,}73\text{m}$

Den Punkt nach $\sqrt{3}$ Meter von O aufstellen