1 a) i) · $D = \mathbb{R} \setminus \{0\}$

· NS: $x - 2 = 0$ für $x = 2$

$x^2 + x + 2 = 0$ hat keine Lösung

· $f'(x) = \frac{1}{4} - (-2) \cdot x^{-3} = 0$ $\rightarrow f'(x) = \frac{1}{4} + \frac{2}{x^3}$

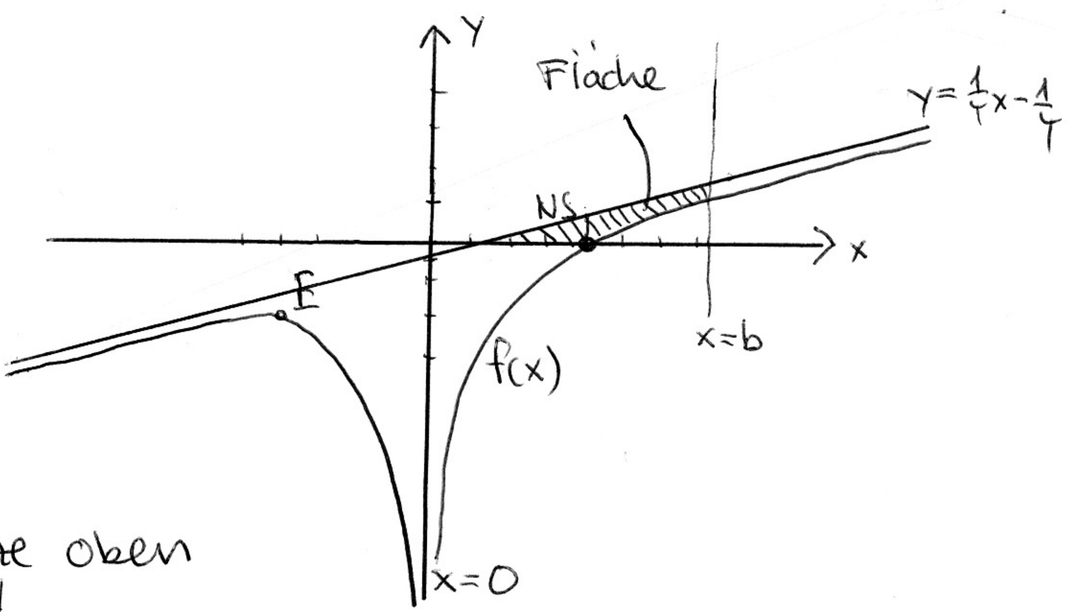
$-\frac{2}{x^3} = \frac{1}{4}$

$x^3 = -8 \Rightarrow x = -2 \Rightarrow \underline{E(-2|-1)}$

· $f''(x) = 2 \cdot (-3) \cdot x^{-4} = 0$ hat keine Lösung $\Rightarrow$ keine WP

ii) $x = 0$ und $y = \frac{1}{4}x - \frac{1}{4}$ (da $-\frac{1}{x^2} \longrightarrow 0$ für $x \to \pm\infty$)

iii) iv)



b) i) siehe Skizze oben

b) ii)

$\Rightarrow A_\Delta = \frac{1}{8}$, Rest: $\int_2^b$ Gerade − Funktion dx

$\Rightarrow A = \frac{1}{8} + \int_2^b \frac{1}{4}x - \frac{1}{4} - \left(\frac{1}{4}x - \frac{1}{4} - \frac{1}{x^2}\right) dx = \frac{1}{8} + \int_2^b \frac{1}{x^2}\, dx = \frac{1}{8} + \int_2^b x^{-2}\, dx$

$= \frac{1}{8} + \left[\frac{x^{-1}}{-1}\right]_2^b = \frac{1}{8} + \left[-\frac{1}{x}\right]_2^b = \frac{1}{8} - \frac{1}{b} + \frac{1}{2} = \underline{\underline{\frac{5}{8} - \frac{1}{b}}}$

iii) $\lim\limits_{b \to \infty} \frac{5}{8} - \frac{1}{b} = \underline{\underline{\frac{5}{8}}}$

c) $y = mx + q$; $m = f'(2) = \frac{1}{2}$, da $(2|0) \in$ Tangente $\Rightarrow q = -1$

Schneide $y = \frac{1}{2}x - 1$ mit $f(x)$ $\Rightarrow$ Schnittpunkt $\underline{\underline{(-1|-1{,}5)}}$

② a) $\pm d(S,E) = \dfrac{2-2\cdot 8-6}{\sqrt{1^2+(-2)^2}} = -\underline{\underline{8,94}}$

$d(M,S) = |\overrightarrow{MS}| = \left|\begin{pmatrix}0\\4\\-8\end{pmatrix}\right| = \sqrt{16+64} = \underline{\underline{8,94}}$ ✓

b) $h: \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1\\2\\8\end{pmatrix} + t\begin{pmatrix}3\\0\\-12\end{pmatrix} \quad \cap E$

↳ $(2+0\cdot t) - 2(8-12t) - 6 = 0 \Rightarrow t = \frac{5}{6}$

$\Rightarrow \underline{\underline{D(3.5|2|-2)}}$

c) $\left(|\overline{ST}| = \left|\begin{pmatrix}3\\0\\-12\end{pmatrix}\right| = \sqrt{9+144} = \sqrt{153} \right.$

$\left. |\overline{SD}| = \left|\begin{pmatrix}2.5\\0\\-10\end{pmatrix}\right| = \sqrt{6.25+100} = \sqrt{106.25} \right)$

$\dfrac{\sqrt{106.25}}{\sqrt{153}} = \dfrac{5}{6} \Rightarrow \underline{\underline{\overline{SD} : \overline{DT} = 5:1}}$

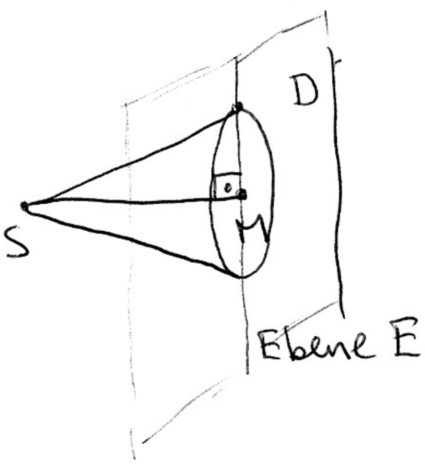
(da in b) $t = \frac{5}{6}$)

d) $\sin(\alpha) = \dfrac{\begin{pmatrix}0\\1\\-2\end{pmatrix} \cdot \begin{pmatrix}3\\0\\-12\end{pmatrix}}{\sqrt{5}\,\sqrt{153}} = \dfrac{24}{\sqrt{5}\,\sqrt{153}} \Rightarrow \underline{\underline{\alpha = 60,19°}}$

e) $A = \frac{1}{2} \cdot |\overrightarrow{SM} \times \overrightarrow{SD}| = \frac{1}{2}\left|\begin{pmatrix}0\\4\\-8\end{pmatrix} \times \begin{pmatrix}2,5\\0\\-10\end{pmatrix}\right| = \frac{1}{2}\left|\begin{pmatrix}-40\\-20\\-10\end{pmatrix}\right| = \frac{1}{2}\sqrt{40^2+20^2+10^2}$

$= \frac{1}{2} \cdot \sqrt{2100} = \underline{\underline{22,91}}$

f)



$h = |\overrightarrow{SM}| \quad ; \quad r = |\overrightarrow{MD}|$

$|\overrightarrow{MD}| = \left|\begin{pmatrix}3.5\\-4\\-2\end{pmatrix}\right| = \sqrt{26,25}$

$|\overrightarrow{SM}| = \left|\begin{pmatrix}0\\-4\\8\end{pmatrix}\right| = \sqrt{80}$

$\Rightarrow V = \frac{\pi}{3} r^2 h = \underline{\underline{245,87}}$

③ a) A: $p = \frac{1}{3}$

B: $p = \frac{1}{3}$

C: $p = \frac{1}{2}$

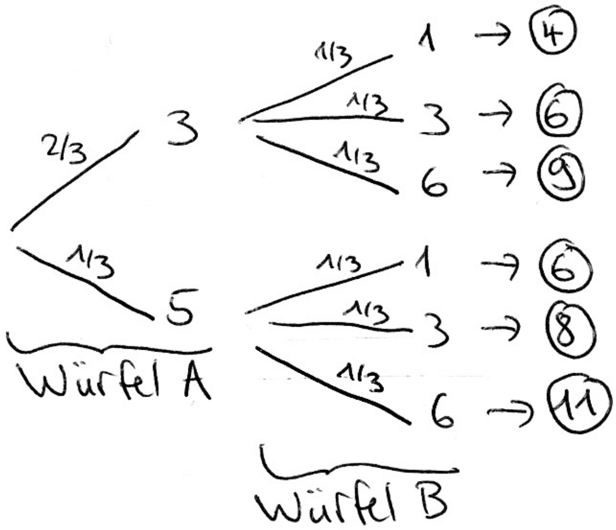
b) Durchschnittliche Wurfhöhe:

$A = \frac{2}{3} \cdot (3) + \frac{1}{3} \cdot 5 = \frac{11}{3}$ $\quad B = \frac{1}{3} \cdot 1 + \frac{1}{3} \cdot 3 + \frac{1}{3} \cdot 6 = \frac{10}{3}$

$C = \frac{1}{6} \cdot (1+2+3+4+5+6) = 3.5$

$\Rightarrow$ $\underline{\underline{\text{Würfel A}}}$

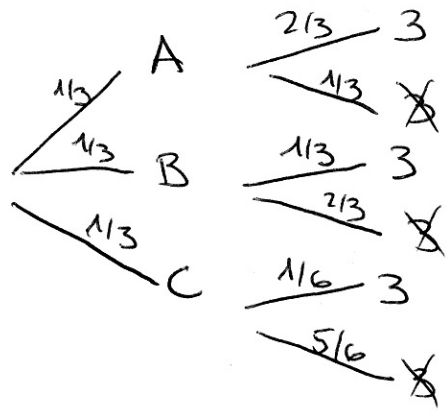
c) i)



$\Rightarrow$ ii) $p = \frac{2}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{3}$

$p = \underline{\underline{\frac{4}{9}}} = 0.\overline{4}$

d)



$P(A \mid \text{Drei}) = \frac{1/3 \cdot 2/3}{\frac{1}{3} \cdot \frac{2}{3} + \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{6}}$

$= \underline{\underline{\frac{4}{7}}} \approx \underline{\underline{0.57}}$

e) Summe grösser 90 bei mindesten 8 Fünfen (91)

$\Rightarrow \sum_{k=8}^{25} B(25, \frac{1}{3}, k) = \underline{\underline{0{,}630 = 63\%}}$

↑ P(Fünf)

4 a) i) zu Beginn: $f(6) = \underline{\underline{12{,}48°C}}$

am Ende: $f(21) = \underline{\underline{19{,}23°C}}$

Max: $f'(t) = 0$ für $\underline{\underline{t = 16}} \Rightarrow f(t) = \underline{\underline{26{,}48°C}}$

Min: am Rand bei $\underline{\underline{t = 6}} \Rightarrow f(t) = \underline{\underline{12{,}48°C}}$

ii) schneide $f(t)$ mit $y = 20$

$\Rightarrow 20 = -0{,}01t^3 + 0{,}24t^2 + 6$

$\Rightarrow t = 10,\ 20{,}74$

$\Rightarrow \underline{\underline{10{,}75\,h = 10h\ 45\,min}}$

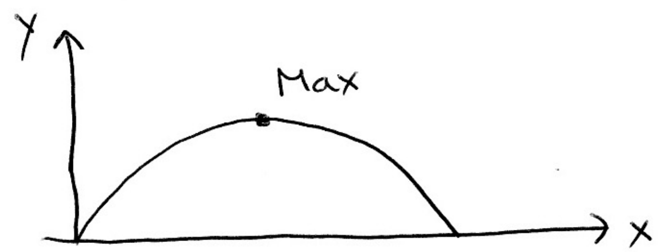
iii) $\frac{1}{15}\int_6^{21} f(t)\,dt = \frac{1}{15}\left[-0{,}01\frac{t^4}{4} + 0{,}24\frac{t^3}{3} + 6t\right]_6^{21} = \underline{\underline{22{,}04°}}$

b) i) $v_0 = 21\,\frac{m}{s} = 75{,}6\,\frac{km}{h}$

ii) $y(x) = \underbrace{\tan(45°)}_{1} \cdot x - \frac{9{,}81}{2 \cdot 21^2 \cdot \underbrace{\cos^2 45°}_{1/2}} \cdot x^2$

$$\boxed{y(x) = x - \frac{9{,}81}{21^2}x^2}$$

$\Rightarrow$



• maximale Höhe

$y'(x) = 0$ für $x = 22{,}48$

$\Rightarrow \underline{\underline{y_{max} = 11{,}24\,m}}$

• nach 35m

$\underline{\underline{y(35) = 7{,}75\,m}}$

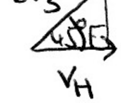
• Winkel nach 35m

$y'(35) = -0{,}557$

$\Rightarrow \alpha = \arctan(-0{,}557) = \underline{\underline{-29{,}12°}}$

Dauer des Flugs:

Horizontale Geschwindigkeit:



$v_H = v_0 \cdot \cos 45° = \frac{21}{\sqrt{2}}\,\frac{m}{s}$

$t = \frac{s}{v} = \frac{35}{\left(\frac{21}{\sqrt{2}}\right)} = \underline{\underline{2{,}36\,s}}$

⑤ a) $\pm d = \dfrac{-12+10\cdot 2.5+7.4}{\sqrt{1^2+10^2}} = \underline{\underline{2{,}03}}$

b) i) $f: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 38 \\ -12 \\ 2.5 \end{pmatrix} + t\begin{pmatrix} 5 \\ -2 \\ -10 \end{pmatrix} \quad \cap E$

$\hookrightarrow \quad (-12-2t)+10(2.5-10t)+7.4=0$

$\Rightarrow \quad t = 0.2$

$\Rightarrow \quad \underline{\underline{L(39|-12.4|0.5)}}$

ii) legt also den Vektor $\begin{pmatrix} 1 \\ -0.4 \\ -2 \end{pmatrix}$ zurück $\rightarrow$ Länge 2,27 km

mit $30\,\frac{km}{h}$ $\rightarrow t = \frac{s}{v} = \dfrac{2{,}27}{30\,\frac{km}{h}} = \underline{\underline{4\,min\ 33\,sec}}$

d) $d = \dfrac{|\vec{AB} \times \vec{u}_f|}{|\vec{u}_f|} = \dfrac{\left|\begin{pmatrix} 2 \\ 0 \\ -1.7 \end{pmatrix} \times \begin{pmatrix} 5 \\ -2 \\ -10 \end{pmatrix}\right|}{\left|\begin{pmatrix} 5 \\ -2 \\ -10 \end{pmatrix}\right|} = \dfrac{\left|\begin{pmatrix} -3.4 \\ 11.5 \\ -4 \end{pmatrix}\right|}{\sqrt{129}}$

$d = \dfrac{\sqrt{159{,}81}}{\sqrt{129}} = \underline{\underline{1{,}11}}$

e) Ebene F mit $s \subset F$ und $f \parallel F$:

$\vec{n}_F = \begin{pmatrix} 4 \\ 9 \\ 0.1 \end{pmatrix} \times \begin{pmatrix} 5 \\ -2 \\ -10 \end{pmatrix} = \begin{pmatrix} -89{,}8 \\ 40.5 \\ -53 \end{pmatrix}$

Ansatz $F: -89{,}8x + 40.5y - 53z + D = 0 \quad \Big|\ \hookrightarrow (4|20|0.7)$

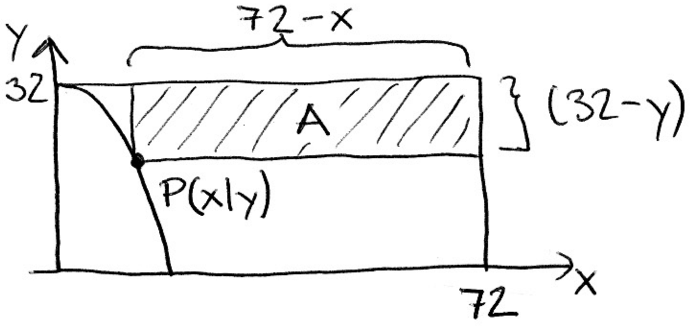
$\Rightarrow \quad D = 3178{,}3$

$d(f,s) = \pm d(A,F) = \dfrac{-89{,}8\cdot(38)+40.5(-12)-53(2.5)+3178{,}3}{\sqrt{(-89{,}8)^2+40.5^2+(-53)^2}}$

$= -7{,}62$

also $\underline{\underline{7{,}62}}$

6 a)



Wähle P auf Parabel $\Rightarrow P(x \mid \overbrace{-\frac{1}{8}x^2+32}^{\text{y-Koordinate}})$

ZF: $A(x) = (72-x)\cdot(32-y) \quad \mid y = -\frac{1}{8}x^2+32$

$A(x) = (72-x)\cdot(32-(-\frac{1}{8}x^2+32))$

$A(x) = 9x^2 - \frac{1}{8}x^3$

Maximum: $A'(x) = 18x - \frac{3}{8}x^2 = 0$ für $x = 0$ (Min)

für $x = 48$ ($\notin D$)

Definitionsbereich: $x \in [0, 16]$ ↖ Nullstelle Parabel

$\Rightarrow$ testen $A(0)=0$ (Min) ; $A(16) = 1792$

$\Rightarrow$ Bei $x = 16$; Seitenlängen 56 und 32

b) i)

S	10× Fünf	mind 8× Drei	genau 5× Drei	sonst
X	10'000	100	50	−5
P	$B(10; \frac{1}{3}; 10) = 0{,}000017$	$\sum_{k=8}^{10} B(10; \frac{2}{3}; k) = 0{,}299$	$B(10; \frac{1}{3}; 5) = 0{,}1365$	Gegenwk $1 - 0{,}000017 - 0{,}299 - 0{,}1365 = 0{,}564$

$\Rightarrow E(X) = 0{,}000017 \cdot 10'000 + 0{,}299 \cdot 100 + \ldots = 34{,}09$ Fr

$s = \sqrt{(0{,}000017 \cdot 10000^2 + 0{,}299 \cdot 100^2 + \ldots) - 34{,}09^2} =$

$\sqrt{5040{,}44 - 34{,}09^2} = \sqrt{3878{,}29} = 62{,}28$ Fr

ii) $\sum_{k=4}^{n} B(n, \frac{1}{3}; k) \geq 0.99$

n	18	20	21	22
P	89,8%	93,96%	95,4%	96,5%

$\Rightarrow$ Mind. 21 Mal