Final Exam Mathematics 2016

Profiles E, I, L, M, S, W, Z

Candidate

Name: .... Class: ....

The following rules apply:

- The duration of the exam is 4 hours.
- A maximum of 48 points can be achieved.
- The solution process for all problems must be clear and complete. Show your use of the calculator (TI-nspire cx CAS). The memory of the calculator has to be cleared before the exam.
- Part 1: You may solve problems (1), (2), (3) and (4) with aid of your calculator and the mathematics formulary (Adrian Wetzel).
- Part 2: Once you hand in your calculator, you will receive problems (5) and (6) and solve them with aid of the mathematics formulary only. You may continue solving the other problems. You may not change back. At the end, you hand in all answers.

Class / Examiner / Expert

Class **Examiner** **Expert**

Grading Scale

Problem	(1)	(2)	(3)	(4)	(5)	(6)	Total
Possible Points	9	5	6	10	10	8	48
Achieved Points							

Final Grade = $\frac{\text{points made}}{41} \cdot 5 + 1$, rounded to half a mark →

Final Exam Mathematics 2016

Profiles E, I, L, M, S, W, Z

Candidate

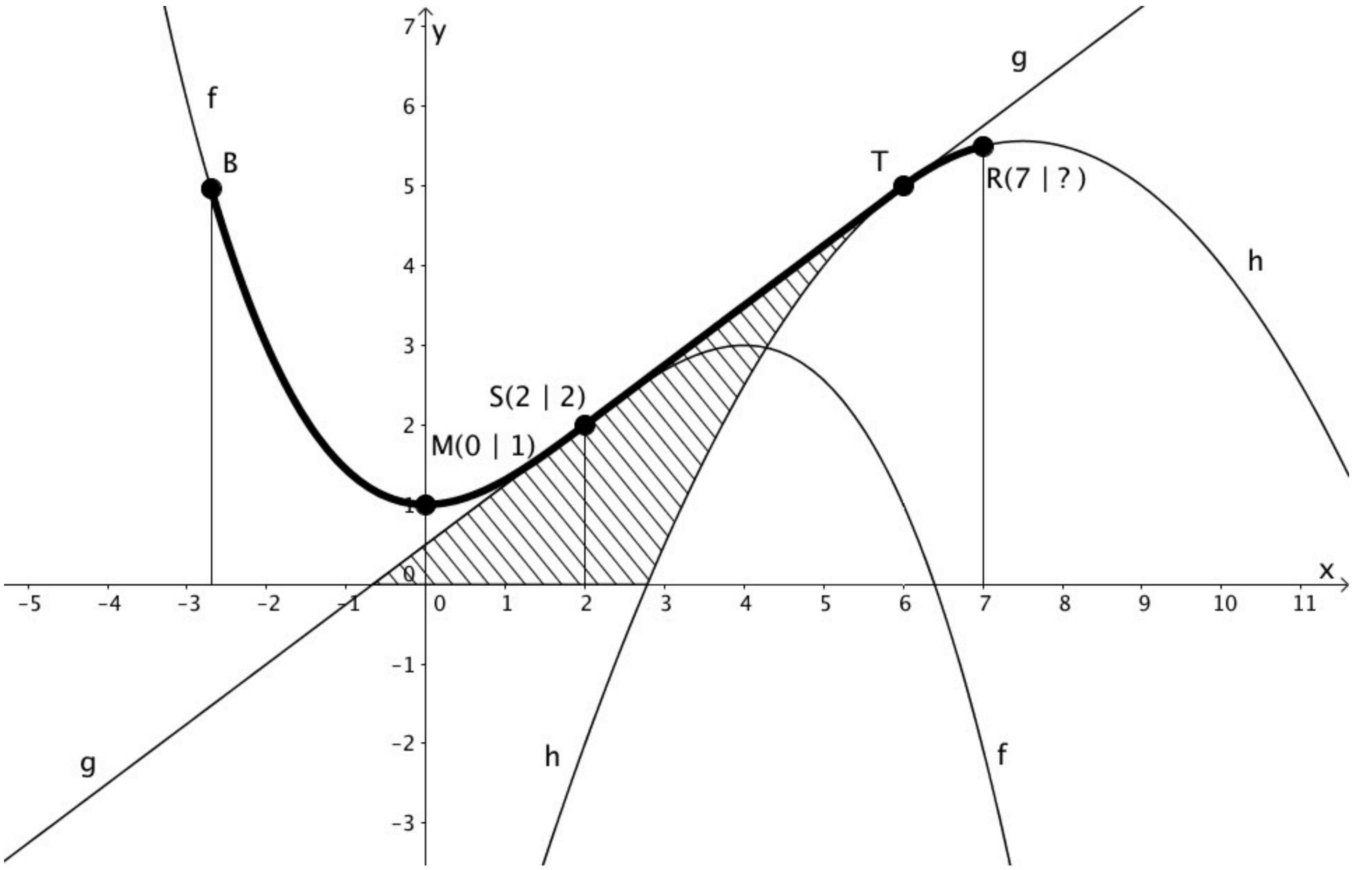
Name: .... Class: ....

Part 1: with calculator

Problem 1 (with calculator) *9 Points*

Function g is given by the following equation: $g(x) = \frac{3}{4} \cdot x + \frac{1}{2}$

Function h is given by the following equation: $h(x) = \frac{-1}{4} \cdot x^2 + \frac{15}{4} \cdot x - \frac{17}{2}$



1.1 Calculate the coordinates of the intersection point T between the graphs of g and h, and show that the graph of g is tangent to the graph of h in point T. *(2.5 P.)*

1.2 Calculate the area bounded by the x-axis, the line g and the parabola h (the shaded area in the figure above). *(2.5 P.)*

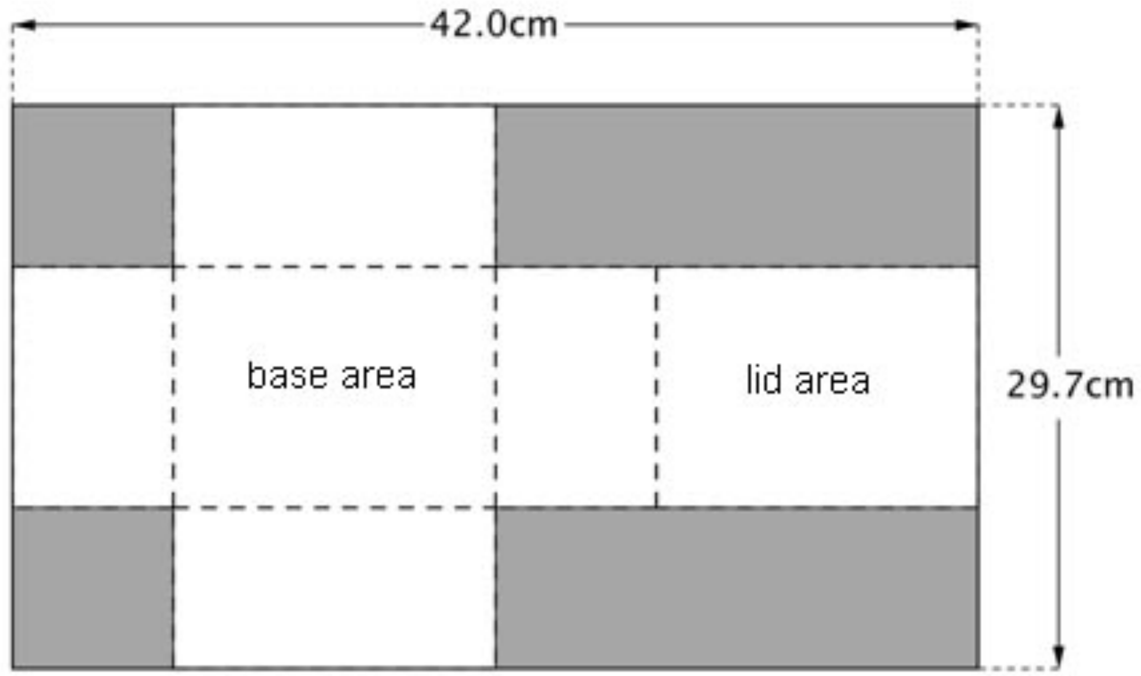
The (bold) line connecting the points B-M-S-T-R of the graphs f, g and h is rotated about the x-axis. This represents the outside edge of a drinking cup.
The section bounded by B-M-S is made of solid glass and represents the foot of the drinking cup.
The section bounded by S-T-R can hold a liquid beverage.

1.3 How much liquid (volume) can be filled maximally into the section of the drinking cup bounded by S-T-R?
Remark: The thickness of the glass is disregarded. *(1.5 P.)*

1.4 The function f is a polynomial of degree 3 ($y = f(x) = a \cdot x^3 + b \cdot x^2 + c \cdot x + d$). The point $M(0,1)$ is the local minimum of the graph of f. The graph of g is tangent to the graph of f in $S(2,2)$.
Find a, b, c and d. *(2.5 P.)*

Problem 2 (with calculator) *5 Points*

A box with lid is made from a rectangular piece of cardboard, with sides 42.0 cm und 29.7 cm by cutting out the shaded areas and folding along the dotted lines shown in the figure below.



2.1 Calculate the volume of the box if the height of the box is 4 cm. *(1.5 P.)*

2.2 A box of maximal volume should be designed. What are the dimensions - height, length and width – of the box? In addition, find the maximal volume. *(3.5 P.)*

Problem 3 (with calculator) *6 Points*

A ball is dropped from a height of 2.5 m. Each time the ball hits the floor, it bounces back to 85 % of its previous height.

3.1 How high will the ball bounce **directly after** the 15^{th} ground contact. *(1 P.)*

3.2 How far will the ball have travelled when it hits the ground for the 15^{th} time?*(2 P.)*

3.3 What total distance does the ball travel if it continues to bounce infinitely as described? *(2 P.)*

3.4 What percentage of its height is reached by another ball after each bounce, if the total distance travelled by this ball is exactly 50 m (the ball is dropped from the same initial height and it bounces infinitely)? *(1 P.)*

Problem 4 (with calculator) *10 points*

According to TNW (Tarifverbund Nordwestschweiz), the proportion of “fare dodgers” - these are passengers which do not hold a valid ticket – among all passengers on all of their 12 tramlines is equal to 3%.

At a ticket inspection to catch fare dodgers, 2 tramlines are selected at random.

4.1 How many options exist to choose 2 different tramlines of the TNW tram network if the order of the chosen tramlines is disregarded. *(1 P.)*

4.2 How many options remain if exactly one of the chosen tramlines has to be of tramlines 10, 11 or 17. *(1 P.)*

Two ticket inspectors board tramline 10 at the tram stop “Bahnhof SBB”. They check all 25 passengers in the tram. At the stop “Bankverein” they change trams and board tram 14 where they check another 18 passengers.

4.3 Calculate the probability that in this control the ticket inspectors catch exactly 2 fare dodgers. *(1.5 P.)*

4.4 Calculate the probability that in this control the ticket inspectors catch at least one fare dodger. *(1 P.)*

4.5 Find the probability that, only after having boarded tram 14, the ticket inspectors catch at least one fare dodger. *(1.5 P.)*

4.6 Determine the number of passengers that have to be checked for the probability of catching at least one fare dodger to be 90%. *(1 P.)*

Detailed analyses show that the fare dodger rate differs on each tramline. On tramline 10 there are 2%, whereas on tramline 14 there are 4% fare dodgers.

4.7 Using these detailed numbers, find the probability that in the control described above (see 4.4) at least one fare dodger is caught. *(1.5 P.)*

4.8 Further analysis revealed that of the 4% fare dodgers on tramline 14 the proportion of male fare dodgers is 70%. This is true even though men and women use tramline 14 with equal frequency. The first passenger that was checked on tram 14 was a man. What is the probability of this man being a fare dodger? *(1.5 P.)*

Final Exam Mathematics 2016

Profiles E, I, L, M, S, W, Z

Candidate

Name: .... Class: ....

Part 2: without calculator

Problem 5 (without calculator)	*10 Points*

Given the three points A(2,2,0), B(4,0,1) and C(0,1,2).

5.1 Points A and B lie on plane E: $x+2y+2z-6=0$.
Prove by way of calculation that point C lies on plane E as well. *(1 P.)*

5.2 Calculate the distance between point D(1,1,1) and plane E. *(1.5 P.)*

5.3 Indicate the normal vector $\vec{n}$ of plane E.
Show by way of calculation that it is perpendicular to vector $\vec{AB}$. *(1.5 P.)*

5.4 A ray of light starts in point P(7,2,2) and is reflected in point S(2,1,1) on plane E.
By way of calculation, determine the parametric equation of the line g on which the reflected light beam travels. *(4 P.)*

5.5 Plane E and the three coordinate planes bound a three-sided pyramid.
Calculate the volume of the pyramid. *(2 P.)*

Problem 6 (without calculator) *8 points*

In each statement 6.1 to 6.13 you need to decide if it is TRUE or FALSE.
In statements 6.14 to 6.16 you need to tick the function that meets the indicated condition.
Evaluation:

Correct answer:	+0.5 p
The first 3 incorrect answers:	0 p
Further Incorrect answers:	-0.5 p
No answer:	0 p
Minimum possible points:	0 p

		TRUE	FALSE
6.1	Polynomial functions of degree 3 always have 3 different zeros.	❑	❑
6.2	Polynomial functions of degree 3 may but do not need to have an inflection point.	❑	❑
6.3	The y-intercept of the function $f(x) = -2x^3 - 4x^2 + 0.5x - 1$ is 1.	❑	❑
6.4	The function $f(x) = 2x^2$ is monotonically increasing.	❑	❑
6.5	All saddle points of a graph are also inflection points.	❑	❑
6.6	The 10th derivative of e^{2x} is $2^{10} \cdot e^{2x}$ (e being Euler's number).	❑	❑
6.7	The 2016th derivative of $\sin x$ is $-\sin x$.	❑	❑
6.8	For each arithmetic sequence $a_1, a_2, a_3, \ldots$, the sequence $2^{a_1}, 2^{a_2}, 2^{a_3}, \ldots$ is also an arithmetic sequence.	❑	❑
6.9	If the elements a_1 and a_4 of an integer arithmetic sequence are even, then the element a_{17} is even as well.	❑	❑
6.10	In a geometric sequence $a_1, a_2, a_3, \ldots$ and $a_2 \neq 0$, $\frac{a_5}{a_2}$ is always positive.	❑	❑
6.11	If $\vec{a}$ and $\vec{b}$ point in the same direction, then: $\vec{a} = \vec{b}$	❑	❑
6.12	$(\vec{a} \cdot \vec{b}) \cdot \vec{c} = \vec{a} \cdot (\vec{b} \cdot \vec{c})$	❑	❑
6.13	$\vec{a} \times (\vec{b} \times \vec{c})$ is a vector.	❑	❑

Problem 6 is continued on the next page...

Problem 6 (without calculator) / continuation

Given the equations of the three rational functions:

$$f(x) = \frac{3 \cdot x^2}{x^3 + 1} \quad ; \quad g(x) = \frac{x^3 - x^2 + 2 \cdot x}{(x-3) \cdot (x-4)} \quad ; \quad h(x) = \frac{(4 \cdot x - 8) \cdot (x+2)}{x^2 + 1}$$

In each of the following problems, one answer is true.

Tick the correct answer (out of the three options).

		f(x)	g(x)	h(x)
6.14	Which function has the largest number of different real zeros?	❑	❑	❑
6.15	Which function does not have a vertical asymptote?	❑	❑	❑
6.16	Of which function is the x-axis a horizontal asymptote?	❑	❑	❑

We wish you much success!