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Matura Exams 2014 – Mathematics

Classes: 4(A)W, 4GL, 4IM, 4IS, 4LZ, 4Sb, 4SW, 4Wb, 5KSW

Duration of Exam:	4 hours
Remark:	Start each question on a fresh sheet of paper.
Additional material:	TI-Nspire CAS Calculator in *Press-to-Test* Mode
	Formelsammlung (*Fundamentum Mathematik und Physik*) or Formula Book (*English*)
	Dictionary: German/English on the Invigilator's Desk

Question 1: Vector Geometry

The point $M(2|5|-1)$ and the line $d:\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}-8\\4\\3\end{pmatrix}+t\cdot\begin{pmatrix}2\\1\\0\end{pmatrix}, t\in\mathbb{R}$, are given.

(a) The point $P(x_P|7|z_P)$ lies in the line d. Calculate x_P and z_P and hence show that the distance $\overline{MP}$ is equal to 6 units. [1,5]

(b) A different point in the line d, called Q, also lies at a distance of 6 units from the point M. Calculate the coordinates of point Q. [1,5]

The plane Π_1 contains both the line d and the point M.

(c) Show that Π_1 may be represented by the Cartesian equation: $x-2y+2z+10=0$. [2]

The upright cone, C_1, has the following properties:

- The circle which forms the base of C_1 lies in the plane Π_1. This circle has its centre point at M and passes through the point P.
- Also the line which passes through the point P and the cone's summit point, S, has the vector équation:

$$l_{PS}:\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}12\\-9\\7\end{pmatrix}+t\cdot\begin{pmatrix}7\\-8\\2\end{pmatrix}.$$

(d) Calculate the angle α between line PS and plane Π_1. [1,5]

(e) Calculate the coordinates of the summit point, S, and the height, h, of the cone C_1. [3]

(f) Find a Cartesian equation for a plane Π_2 which cuts the cone C_1 into two solids of equal volume. [2,5]

Question 2: Calculus

The family of functions $f_k(x) = k \cdot x^3 - \frac{3}{2}x^2 + 4x$, with $k \in \mathbb{R}$, $k \neq 0$, and the line g with equation $g(x) = -\frac{1}{2}x$ are given.

(a) Find the value of the parameter k, so that the inflexion point on the graph $f_k(x)$ lies on the x-axis. [2]

For the remaining parts of this question, you should use $k = \frac{1}{8}$. Also, the graph of the function $f_{\frac{1}{8}}(x)$ will be called K.

(b) Calculate the coordinates of the Zero Points, Maximum and Minimum points of K. [2]

(c) Show that the line g and the curve K touch each other. [2]

(d) A bounded region, R_1, is formed by K and line g. Calculate the area of the region R_1. [2]

(e) The bounded region, R_2, is formed by the x-axis, K and line g. Calculate the volume of the solid formed when region R_2 is rotated $360°$ around the x-axis. [2]

(f) There exists a second line, h (different from line g), which passes through the origin and together with K forms exactly one bounded region, R_3. Find the gradient of line h. [2]

Question 3: Calculus

The two functions $f(x) = (1 + 2x)e^{-0,5x}$ and $g(x) = e^{-0,5x}$ are given.
The graph of $f(x)$ will be known as $K_f$and that of $g(x)$ as K_g.

(a) Show *by hand* that $f'(x) = (1{,}5 - x)e^{-0,5x}$ and that $f''(x) = (0{,}5x - 1{,}75)e^{-0,5x}$. [2]

(b) Calculate the full coordinates of any maximum points or inflexion points for K_f. [2]

(c) At what (acute) angle do K_f and K_g meet? [2,5]

(d) The line with equation $x = k$, where $k > 0$, cuts the curves K_f and K_g at the points F and G respectively. Find the value for k which maximizes the distances $\overline{FG}$. Hence, find this maximum distance. [2]

(e) The curve K_g together with the x and y axes create an unbounded region, R. Calculate the area of region R. [1]

(f) The curve K_f intersects the y-axis at the point called P. The tangent line and the normal line to curve K_f at point P cross the x-axis at points T, and N respectively. Calculate the area of the triangle PNT. [2,5]

Question 4: Probability

Beaker (*Becher*) A contains three fair dice (*faire Würfel*). Beaker B contains three unfair, or ”loaded”, dice (*gefälschte Würfel*).
For the loaded dice: the probability of a 6 is $\frac{1}{2}$ and the probability of obtaining each of the other values is $\frac{1}{10}$.

(a) When the three dice in beaker A are thrown together, calculate the probability that:

  i. the numbers 1, 2 and 3 appear together. [2]

  ii. the sum of the three numbers which appear equals 16. [2]

(b) When the three dice in beaker B are thrown together, calculate the probability that the sum of these three numbers equals 15. [3]

(c) One die is picked from each beaker and the two dice are thrown together.

  i. Calculate the probability that the die from beaker A shows a higher number than the number on the die from beaker B. [3]

  ii. In an experiment the same two dice are thrown together many times and the total score is recorded . How many times must this be done for us to be 99,5% sure that at least one double-six will be obtained? [2]

Three Independent Questions

Question 5.1

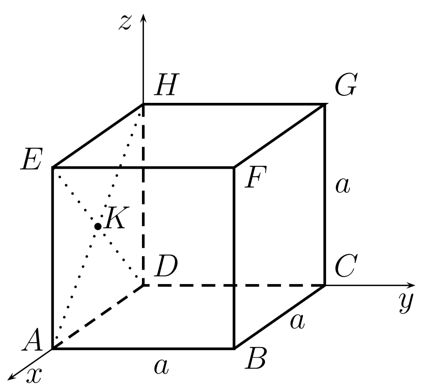
The working speed of office computers is controlled by the frequency of the '*Taktrate*'. On 1st Jan. 1990, this frequency was 50 MHz, then 1 GHz on 1st Jan. 2000. (1 GHz = 1'000 MHz). Assuming that these frequencies change over time according to an exponential model, calculate:

(a) the frequency (Taktrate) we can expect on 1st Jan. 2020. [2]

(b) the number of years required for this frequency to double. [1]

(c) the speed (or rate) of growth of frequency, in GHz per year, on 1st Jan. 2020. [1]

Question 5.2



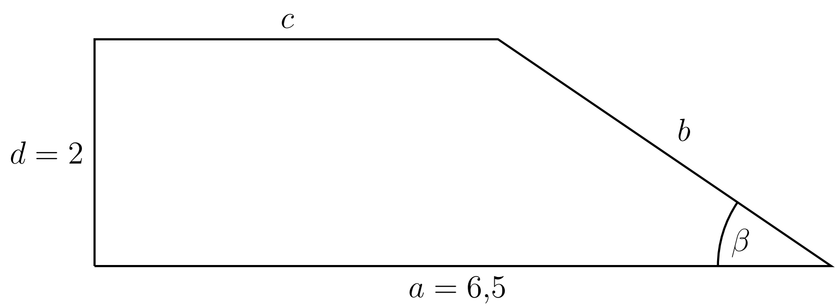
The diagram shows the cube $ABCDEFGH$ with all edges equal to a units. The diagonals $\overline{AH}$ and $\overline{DE}$ intersect at the point labeled K.

Show that the lines DE and BK are perpendicular. [3]

Question 5.3

(a) Calculate the lengths b and c when angle $\beta = 52{,}7°$. [2]

(b) For what angle β are the lengths b and c equal? [3]



With good luck wishes from: Thomas Blott, Rolf Haag, Roman Huber, Andreas Kilberth, Guido Lafranchi, Matthieu Penserini, Mathias Schenker and Alain Zumbiehl.