



Part 1								
1	2	3	4	5	6	7	8	T1
4	3	4	5	3	4	6	4	33

First Name _____

Last Name _____

Part 2								
9	10	11	12	13	14	15	16	T2
3	4	4	3	5	3	5	6	33

Total Points	
Grade	

The “*Mathematics Formulary*“ by Adrian Wetzel is the only permitted aid. The approaches to the solutions should be presented clearly and well-arranged. Incomplete approaches may lead to a deduction of points.

Task 1

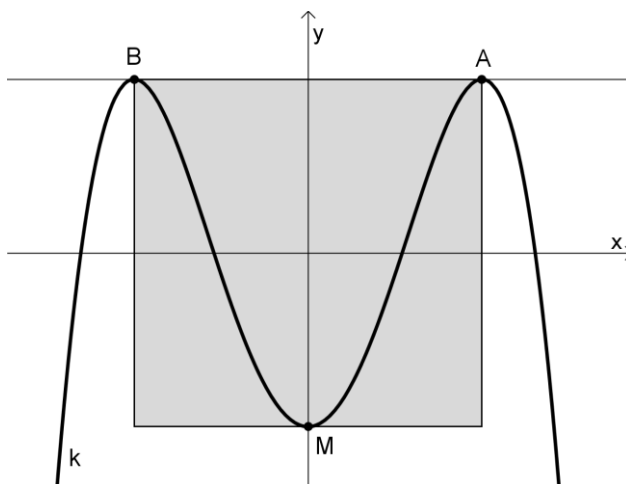
4 points

A square has its center at the origin and side length $s = 2$.

The curve k is the graph of a polynomial of 4th order.

k touches the square at A , B and M (see figure).

Find the equation of k .



Task 2

3 points

The two points $A(0|-2|5)$ and $B(6|4|9)$ and the line $g: \vec{r} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ are given.

Find the coordinates of the point P which lies on the line g and has the same distance to both points A and B .

Task 3

4 points

An arithmetic sequence has difference $d = 4$, the sum of the first n terms is 48, and the $(n + 1)^{\text{st}}$ term is $a_{n+1} = 22$.

Find the first term a_1 and the number of terms n .

Task 4

5 points

The graph of the function $f(x) = -x^2 + c$ ($c > 0$) and the x-axis enclose an area with size A_1 . The graph of the function $g(x) = -x^2 + k \cdot c$ and the x-axis enclose another area with size A_2 . Find k , so that $A_2 = 8 \cdot A_1$.

Task 5

3 points

A dishonest gambler has manipulated a coin. It does not show “heads” or “tails” with the same probability anymore. The probability for “tails” is a bit higher than for “heads”.

If someone throws this coin twice, the probability for two different faces is 0,48. Find the probability that one single throw will show “tails”.

Task 6

4 points

Consider the function $f(x) = \frac{x + \sin(x)}{\cos(x)}$.

Find the equations of the tangents at the points where $x_1 = 0$ and $x_2 = \pi$.

Task 7

6 points

The plane E has the axes intercepts $a = 1$, $b = -3$, $c = -2$.

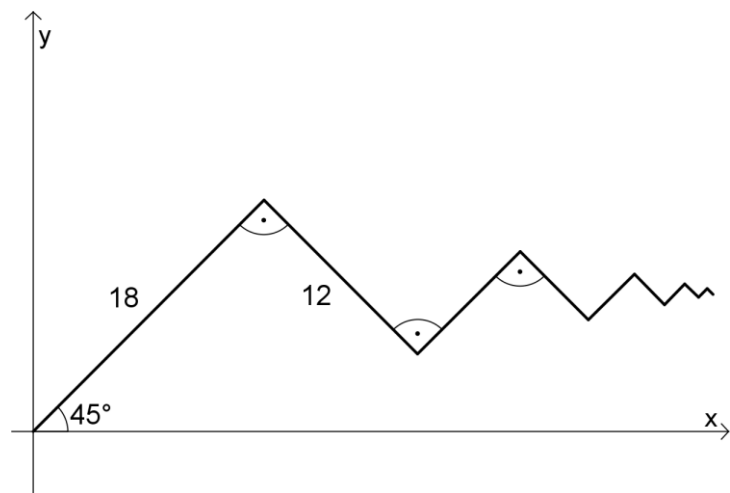
- Find the Cartesian equation of the plane E .
- Find the distance from the point $P(9|1|-1)$ to the plane E .
- A ray of light starting at the point P is reflected by the plane E . Find the point of reflection R , such that the reflected ray passes through the point $Q(9|-19|-4)$.

Task 8

4 points

This zigzag path shown in the figure consists of infinitely many line segments which are perpendicular to each other and whose lengths make up a geometric sequence.

- Find the total length of this infinite zigzag path.
- Find the coordinates of the point to which this zigzag path converges.





Matura Exam 2014
Mathematics
Part 2

Class 4SeWe
Teacher Ae
Duration 90 min.

Part 2								
9	10	11	12	13	14	15	16	T2
3	4	4	3	5	3	5	6	33

First Name _____

Last Name _____

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Task 9

3 points

Consider the points $A(1|2|1)$ and $B(1|4|3)$ and the line g through the points $P(0|1|-1)$ and $Q(1|2|0)$.

- Find a point C on the line g , so that the triangle ABC has an angle of $\alpha = 60^\circ$ at A .
You need to find only one solution.
- Calculate the area of this triangle. If you could not solve a), use $C(2|3|1)$.

Task 10

4 points

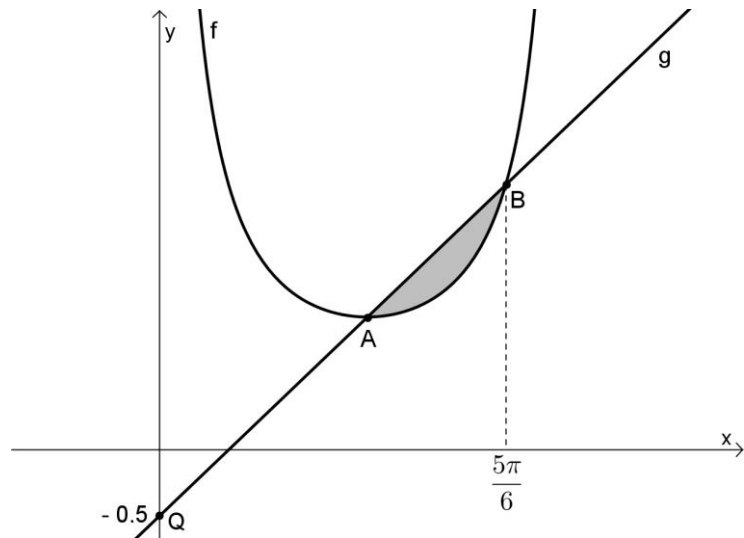
- a) Find the coordinates of the lowest point A on the graph of the function

$$f: y = \frac{1}{\sin(x)} \quad x \in]0, \pi[$$

(You do not need to prove that it is a minimum.)

- b) Show that the line g through A and $Q(0 | -\frac{1}{2})$ intersects the graph of f at $x = \frac{5\pi}{6}$.

- c) The shaded area rotates around the x-axis, generating a solid of revolution. Calculate its volume.



Task 11

4 points

Apu sells hot-dogs in his Kwik-E-Mart. Unfortunately, not all his sausages are fresh. On average, every tenth sausage is rotten.

- a) Homer buys 8 hot-dogs in Apu's Kwik-E-Mart. Find the probability that
- a₁) exactly one hot-dog is rotten.
 - a₂) at least two hot-dogs are rotten.
 - a₃) at most three hot-dogs are rotten.
- b) How many hot-dogs does he need to buy at least, so that the probability to get at least one rotten hot-dog is more than 99%?

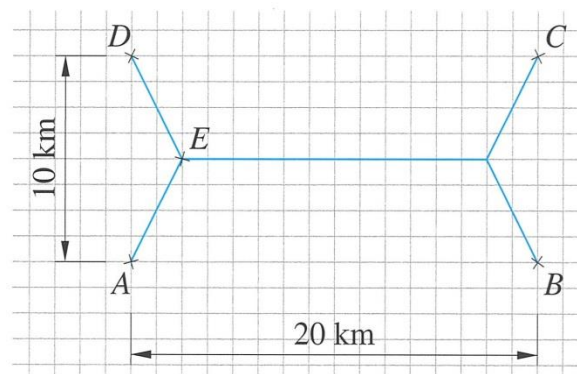
Task 12

3 points

The four points A, B, C and D are to be connected by a rotationally symmetric route network (see figure) with minimum length.

Find the distance from point E to the line AD , and the total length of this optimum network.

Furthermore, prove that it is a minimum.



Task 13

5 points



Matryoshka are Russian dolls, made of wood, which can be placed one inside the other. The bigger dolls can (approximately) be obtained by a central dilation¹, with the scaling centre in the middle of the base of the smallest doll. The heights of these dolls make up a geometric sequence.

Consider a Matryoshka with several parts, where the smallest doll has a height of 1 cm , the volume of the second smallest doll is 5 cm^3 , and the volume of the fourth smallest doll is 20 cm^3 .

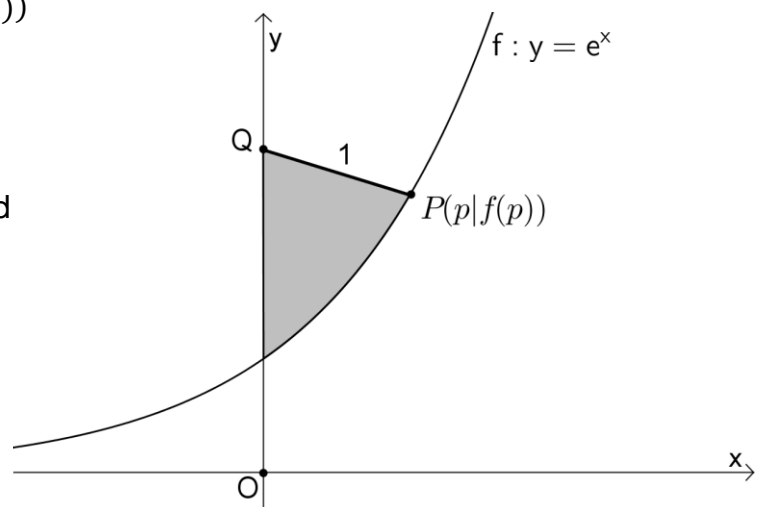
- Find the smallest doll which has a volume of more than 2 litres.
- Find the height of the 9th doll.
- Find the height of the „tower“ with the first 10 dolls placed one above the other.

Task 14

3 points

In this figure, the point $P(p|f(p))$ lies in the first quadrant, on the graph of the function $f: y = e^x$. The line segment PQ has a length of 1.

- Find the size A of the shaded area, depending on the parameter p .
- Find the value of p for which the area A is maximised. (You do not need to prove that it is a maximum).

**Task 15**

5 points

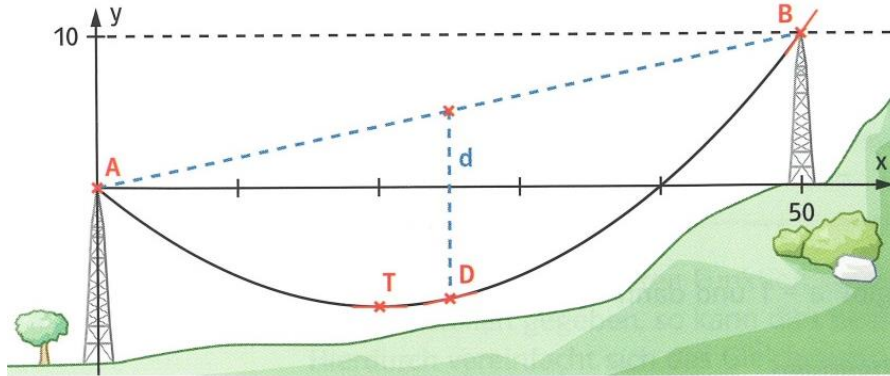
The plane E_1 has equation $E_1: x + 2y - 2z - 19 = 0$. Another plane E_2 passes through the points $A(2|2|0)$, $B(0|2|-2)$ and $C(1|-2|1)$.

- Find the Cartesian equation of the plane E_2 .
- Calculate the angle α of the triangle ABC.
- Find an equation for the line of intersection of these two planes.
- Calculate the acute angle of intersection of these two planes.

¹ Zentrische Streckung

The course of a wire hung between two points (see figure, units in metres) can be described by the so-called „catenary function“:

$$f(x) = a \cdot \left(e^{\frac{x-20}{a}} + e^{\frac{20-x}{a}} \right) + b$$



- a) Find the values of the parameters a and b , so that the wire passes through the points $A(0|0)$ and $B(50|10)$. For the following exercises, round these values to one decimal place.

Hint: First eliminate b from the system of equations, before you use the calculator.

If you could not solve a), use the values $a = 52.8$ and $b = -113.2$ for the following exercises.

In b) and c) you do not need to prove that it is an extremum.

- b) Find the x-coordinate of the lowest point T.
 c) Find the length of the maximum sag² d (see figure).
 d) Find the angle which the wire makes up with the horizontal line at point A.