



Kantonsschule Heerbrugg

Matura Exam 2013 **Classes** 4L, 4SeWe, 4GM, 4NP, 4Sa, 4Wa

Mathematics, Part 1 **Teachers** Fi, Ae, Kp, Sp, Bs, Gz

Name: **Duration** 90 minutes

The mathematics formulary used in the lessons is the only permitted aid. The approaches to the solutions should be presented clearly and well-arranged. Incomplete approaches may lead to a deduction of points.

Evaluation

1	2	3	4	5	6	7	Part 1	Total
3	5	6	5	7	5	5	36	70
8	9	10	11	12	13	14	Part 2	Grade
3	4	4	6	7	4	6	34	

Task 1

3 points

Consider the function $g(x) = a \cdot x^3 + b \cdot x + 2$ with these properties:

The point $P(-1|2)$ lies on the graph of this function and the tangent at $x = 1$ is parallel to the line $y = -x$.

- Find a and b .
- For $a = 1/9$ and $b = -3$, find the x -values at which the function $g(x)$ has horizontal tangents.

Task 2

5 points

Consider the function $f(x) = \frac{(x+4)^2}{e^x}$.

Find the domain, the roots, the extrema (and classify them) and the points of inflection. You do not need to calculate the y -coordinates of these points.



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Task 3

6 points

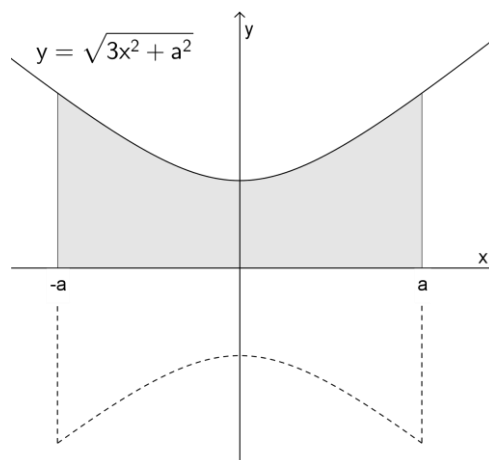
a) The curve $y = \sqrt{3x^2 + a^2}$, where $-a \leq x \leq a$, is rotated around the x-axis. Find the volume of this solid of revolution.

b) Calculate the volume of this solid of revolution, but this time using the approximation formula

$$V = \frac{h}{6}(Q_1 + 4Q_2 + Q_3)$$

where Q_1 is the base area, Q_3 is the top area and Q_2 is the cross-section area in the middle.

c) Find the equation of the parabola that goes through the same points as the curve from task a) at $x = -a$, $x = 0$ and $x = a$. Then, find the volume of revolution of this parabola.



Task 4

5 points

Twelve people, two of whom are smugglers, are travelling on a minibus. At the border crossing point, three people are randomly chosen and controlled.

- Find the probability that the two smugglers are among these controlled people.
- The first controlled person is already revealed as a smuggler. What is the probability that the second smuggler will also be discovered?
- Find the probability that the third controlled person is a smuggler.

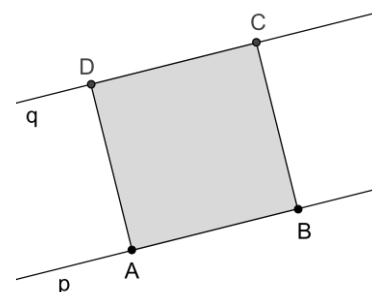
Task 5

7 points

a) The corners of a square $ABCD$ lie on two parallel lines p and q . A and B lie on p , C and D lie on q . Calculate the coordinates of the corners B , C and D , if

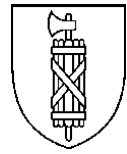
$$A(5|6|1) \text{ and } q: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 3 \end{pmatrix} + t \cdot \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$$

(Only one solution is required.)



b) The square $ABCD$ and the point S make up a straight pyramid (where S is the apex of the pyramid). S lies on the line $g: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -6 \\ 1 \\ 6 \end{pmatrix} + s \cdot \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$. Find the coordinates of S .

(If you could not solve a), use $B(9|10|3)$ and $C(11|6|7)$.)

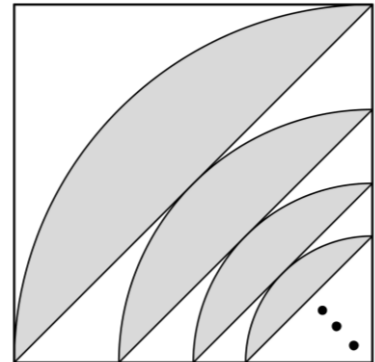


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Task 6

5 points

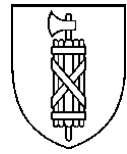
Infinitely many circle segments are inscribed into a square with side length $s = 2 \text{ cm}$ (see figure).
Find the sum of the areas of the infinitely many circle segments.



Task 7

5 points

The first three terms of a geometric sequence have a sum of 35. If the middle term is increased by 25%, these three terms now form an arithmetic sequence. Find the possible solutions for these first three terms.



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Mathematics, Part 2

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Name:

Duration 90 minutes

The mathematics formulary used in the lessons and a CAS-calculator are the only permitted aids. The approaches to the solutions should be presented clearly and well-arranged. Incomplete approaches may lead to a deduction of points.

Task 8

3 points

An athlete is preparing for a run. On the first day he runs 4000 m. For the next 4 weeks, he wants to extend this distance each day by 400 m.

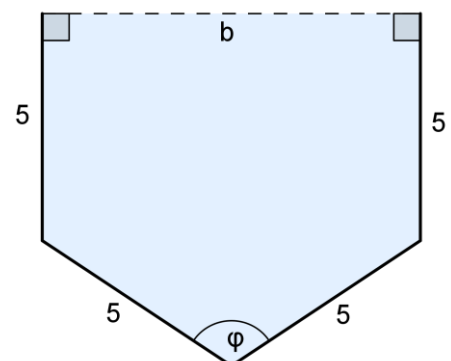
- How much will he run at the last day of his preparation?
- How long would he need to follow this scheme, if he wanted to run a marathon track (42'195 m) in one day? (Answer: whole number)
- How much will he have run in total during these 4 weeks?

Task 9

4 points

A rainwater gutter is to be made out of four iron sheets, each one with a width of 5 cm.

- Find the width of the rainwater gutter (length b , see figure), so that its cross-section area is maximised.
- For this optimum solution, find the angle between the two lower iron sheets at the bottom.





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Task 10

4 points

- a) Find the value of the parameter t , so that the points $A(2t+2|0|0)$, $B\left(0|2+\frac{2}{t}|0\right)$ and $C(0|0|3)$ make up an isosceles triangle (base \overline{BC}).
- b) For $t=2$, find a vector equation of the plane E which contains A and B and is parallel to the z -axis.

Task 11

6 points

In the Israeli lottery, 6 out of 37 balls are drawn.

- a) Find the probability to win the Israeli lottery, i.e. to guess six numbers right.
If you could not solve a), use the probability of $4 \cdot 10^{-7}$ for the further exercises.
- b) What is the probability that the same numbers are drawn in two subsequent drawings?
- c) What is the probability that the numbers 13, 14, 26, 32, 33, 36 are drawn in two subsequent drawings?
- d) What is the probability that the numbers 13, 14, 26, 32, 33, 36 are drawn exactly twice in four drawings?

ausgefallen

Unglaublicher Zufall: In Israel sind am Samstagabend exakt die gleichen sechs Lotto-Zahlen gezogen worden wie im Vormonat. Israelische Medien berichteten, insgesamt 95 Teilnehmer hätten richtig auf die gleichen Zahlen wie bei der Ziehung am 21. September getippt: 13, 14, 26, 32, 33 und 36. Nur drei von ihnen wählten jedoch die richtige Zusatzzahl 2. Sie bekommen jeweils einen Gewinn von 4 Millionen Schekel (1 Million Franken).

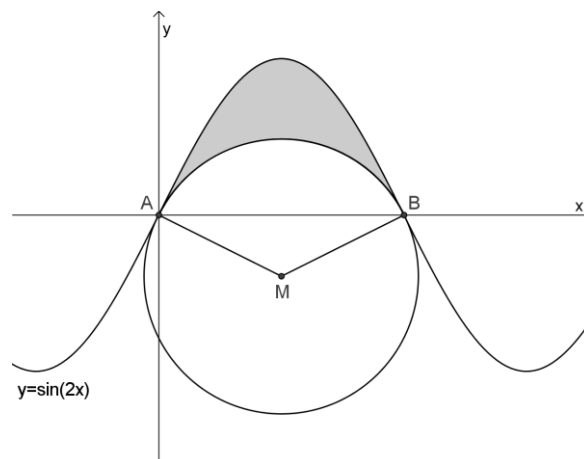
Der Rheintaler, 18. Okt. 2010

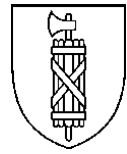
Task 12

7 points

A circle touches the graph of $y = \sin(2x)$ at two subsequent roots (see figure).

- a) Find the coordinates of M , and the angle AMB .
- b) Calculate the size of the shaded area.



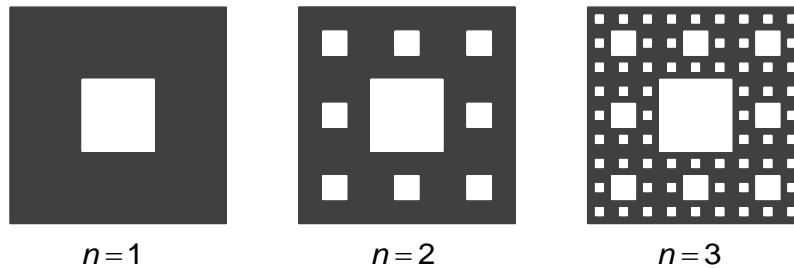


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Task 13

4 points

A figure is constructed in a square with side length one, by cutting out a square with one ninth of the area at the centre. From the eight remaining squares, one ninth is removed at their centres again, then one ninth of the remaining smaller squares, and so on. The figure shows the first three steps of this construction.



- Which percentage of the area was removed in the first three steps?
- How many steps are needed at least so that less than 30% of the area is left?

Task 14

6 points

Consider the line AB through $A(-3|2|-1)$ and $B(-1|-1|-3)$ and the line $h: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 4 \\ 6 \end{pmatrix} + t \cdot \begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}$.

- Show that the lines AB and h are parallel, but not coincident.
- Find the Cartesian equation of the plane E which goes through AB and h .
- Show that B is the perpendicular foot of $S(3|7|-11)$ onto E .
- Rotating the triangle ABS around the axis BS will create a cone. Find the volume of this cone.