Kanton St.Gallen
Bildungsdepartement

Kantonsschule Heerbrugg

Matura Exam 2012	**Class**	**4We**
Mathematics, Part 1	**Teacher**	**Ae**
Name:	**Duration**	**90 minutes**

The "*Mathematics Formulary*" by Adrian Wetzel is the only permitted aid. The approaches to the solutions should be presented clearly and well-arranged. Incomplete approaches may lead to a deduction of points.

Evaluation

1	2	3	4	5	6	7	8	Part 1	Total
3	3	3	4	4	6	6	6	35	70
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**Part 2**	**Grade**
4	3	3	5	4	6	4	6	35	

Task 1 — 3 points

Solve the equation $\log\left(\frac{1}{3}x\right)+\log(3x+10)=2\log(x+1)$.

Task 2

3 points

The velocity $v(\lambda)$ of a wave of wavelength λ in water is

$$v(\lambda) = a \cdot \sqrt{\frac{\lambda}{b} + \frac{c}{\lambda}}$$ (a, b and c are known, positive constants).

What is the wavelength that gives the minimum velocity? (You do not need to show that the extremum is a minimum.)

Task 3

3 points

Given a geometric series with four terms. The sum of the first two terms is 24, the sum of the last two terms is 384. Find the first term.

Task 4 | 4 points

The three planes $A: 3x - 3y + z - 9 = 0$, $B: 5x + 2y - 3z - 29 = 0$ and E (which goes through the origin) have a mutual line of intersection. Find the Cartesian equation of the plane *E*.

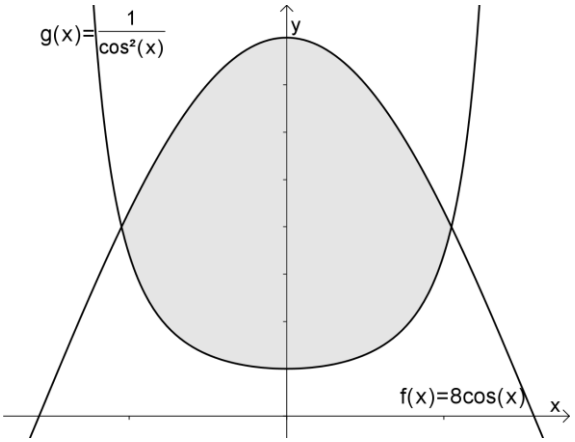
Task 5

4 points

Given the functions

$f(x) = 8 \cdot \cos(x)$ and $g(x) = \dfrac{1}{\cos^2(x)}$.

Calculate the shaded area.



Task 6 6 points

Bart and Milhouse play the following game: They roll a dice by turns. Bart wins if he gets an odd number, and Milhouse wins if he gets an even number. The game stops when someone wins. Bart starts.

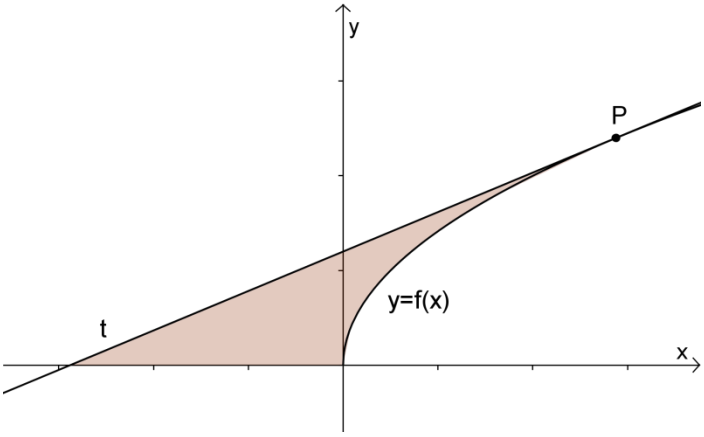
a) The game is not really fair. Bart has a bigger probability of winning.
   Find this probability.

b) To make the game fairer, they decide that Bart only wins if he gets a 5 or 6, and Milhouse wins if he gets a number smaller than k.
   Find k so that the game is fair.

Task 7

6 points

Given the function $f(x) = \sqrt{a \cdot x}$
with $a > 0$.



a) A tangent *t* is laid on the graph of $y = f(x)$ at the point $P\left(a \middle| f(a)\right)$.
Find the equation of *t*.

b) The area enclosed by the curve, the tangent at point P and the x-axis is rotated around the x-axis, generating a solid of revolution. Find its volume.

Task 8 | 6 points

Given the points $A(3|-4|7)$, $B(-5|8|3)$ and the line $g: \vec{r} = \begin{pmatrix} -9 \\ 5 \\ 2 \end{pmatrix} + t \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$.

a) Show that the line through the points *A* and *B* is parallel (but not coincident) to the line *g*.

b) Find the points *C* and *D* on line *g*, so that *ABCD* is an isosceles trapezium with $\overline{AB} = 2 \cdot \overline{CD}$.

Additional space for longer solutions

Please indicate at the task that the solution is continued here.