

**Kantonsschule Heerbrugg**

<b>Matura Exam 2012</b>	<b>Class</b>	<b>4We</b>
<b>Mathematics, Part 1</b>	<b>Teacher</b>	<b>Ae</b>
<b>Name:</b>	<b>Duration</b>	<b>90 minutes</b>

The “*Mathematics Formulary*” by Adrian Wetzol is the only permitted aid. The approaches to the solutions should be presented clearly and well-arranged. Incomplete approaches may lead to a deduction of points.

**Evaluation**

<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>Part 1</b>	<b>Total</b>
3	3	3	4	4	6	6	6	35	70
<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>	<b>15</b>	<b>16</b>	<b>Part 2</b>	<b>Grade</b>
4	3	3	5	4	6	4	6	35	

**Task 1**

3 points

Solve the equation  $\log\left(\frac{1}{3}x\right) + \log(3x+10) = 2\log(x+1)$ .



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**Task 2**

3 points

The velocity  $v(\lambda)$  of a wave of wavelength  $\lambda$  in water is

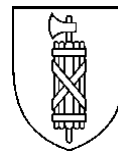
$$v(\lambda) = a \cdot \sqrt{\frac{\lambda}{b} + \frac{c}{\lambda}} \quad (a, b \text{ and } c \text{ are known, positive constants}).$$

What is the wavelength that gives the minimum velocity? (You do not need to show that the extremum is a minimum.)

**Task 3**

3 points

Given a geometric series with four terms. The sum of the first two terms is 24, the sum of the last two terms is 384. Find the first term.



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**Task 4**

4 points

The three planes  $A: 3x - 3y + z - 9 = 0$ ,  $B: 5x + 2y - 3z - 29 = 0$  and  $E$  (which goes through the origin) have a mutual line of intersection. Find the Cartesian equation of the plane  $E$ .



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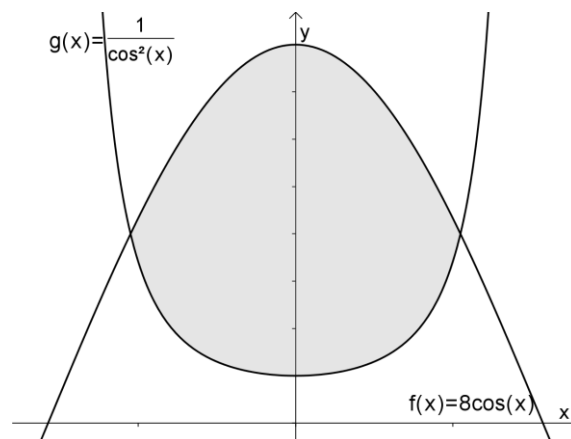
**Task 5**

4 points

Given the functions

$$f(x) = 8 \cdot \cos(x) \quad \text{and} \quad g(x) = \frac{1}{\cos^2(x)}.$$

Calculate the shaded area.





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**Task 6**

6 points

Bart and Milhouse play the following game: They roll a dice by turns. Bart wins if he gets an odd number, and Milhouse wins if he gets an even number. The game stops when someone wins. Bart starts.

- a) The game is not really fair. Bart has a bigger probability of winning.  
Find this probability.
- b) To make the game fairer, they decide that Bart only wins if he gets a 5 or 6, and Milhouse wins if he gets a number smaller than  $k$ .  
Find  $k$  so that the game is fair.



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Task 7

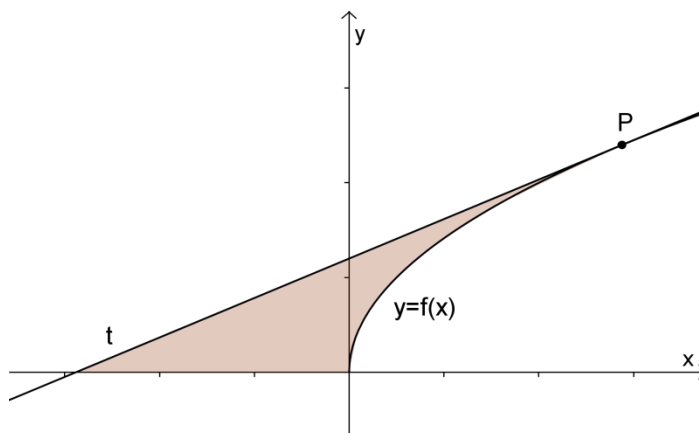
6 points

Given the function  $f(x) = \sqrt{a \cdot x}$   
with  $a > 0$ .

a) A tangent  $t$  is laid on the graph of  
 $y = f(x)$  at the point  $P(a|f(a))$ .  
Find the equation of  $t$ .

b) The area enclosed by the curve,  
the tangent at point P and the

x-axis is rotated around the x-axis, generating a solid of revolution. Find its volume.





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**Task 8**

6 points

Given the points  $A(3|-4|7)$ ,  $B(-5|8|3)$  and the line  $g: \vec{r} = \begin{pmatrix} -9 \\ 5 \\ 2 \end{pmatrix} + t \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ .

- Show that the line through the points  $A$  and  $B$  is parallel (but not coincident) to the line  $g$ .
- Find the points  $C$  and  $D$  on line  $g$ , so that  $ABCD$  is an isosceles trapezium with  $\overline{AB} = 2 \cdot \overline{CD}$ .



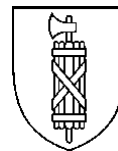
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**Additional space for longer solutions**

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Please indicate at the task that the solution is continued here.





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Mathematics, Part 2

Teacher

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Name:

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90 minutes

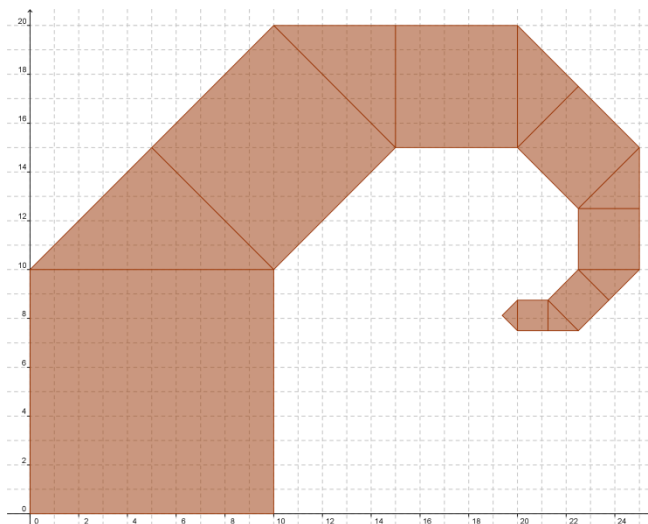
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Task 9

4 points

Consider this figure made of infinitely many squares and isosceles right triangles. The largest square has side length  $s_1 = 10$ .

Calculate the total area.





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**Task 10**

3 points

Given the functions  $f(x) = \sqrt{0.5x - a}$  and  $g(x) = e^{0.5x}$ .

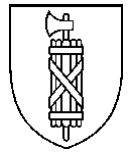
Find parameter  $a$ , so that the graphs of these functions touch each other.

**Task 11**

3 points

The reproduction of germs in cow milk can be described by an exponential function. In  $1 \text{ cm}^3$  cow milk 66'000 germs were counted 3 hours after milking, and 2 hours later there were 1.1 million germs.

- Find the number of germs 6 hours after milking.
- Find the time after which the number of germs is doubled.



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**Task 12**

5 points

Consider the parabola  $y = 1 - x^2$ . In the first quadrant, a point  $P$  is chosen on the parabola. The tangent  $t$  onto the parabola at the point  $P$  forms a triangle with the coordinate axes.

Find the coordinates of  $P$ , so that the area of this triangle is minimised.



**Task 13**

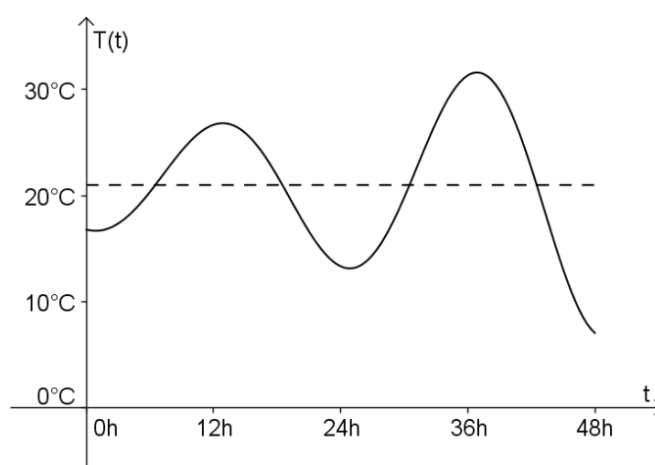
4 points

In the summer break, the temperature was measured for two days. The measuring started at midnight. Due to the cloudless sky on the second day, the fluctuation in temperature increased. The whole temperature profile over these two days can be described mathematically with the function  $T(t)$ :

$$T(t) = 5 \cdot 1.1^{\frac{\pi}{12}(t-6.5)} \cdot \sin\left(\frac{\pi}{12}(t-6.5)\right) + 21$$

$t$ : time in h  
 $T(t)$ : temperature in °C

- a) Find the point of time at which the highest temperature was measured.
- b) Find the point of time at which the temperature decreases the fastest.
- c) Find the average temperature during these two days.



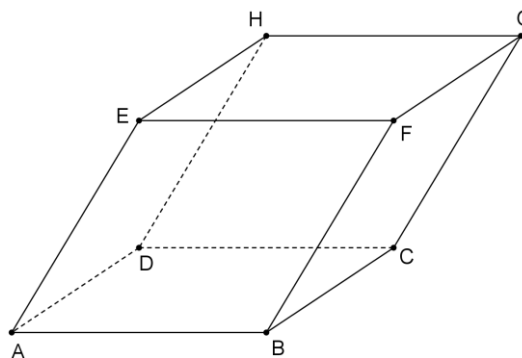


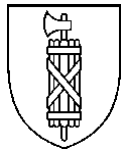
**Task 14**

6 points

$ABCDEFGH$  is a skew prism with the vertices  $A(0|0|0)$ ,  $B(12|2|0)$ ,  $D(1|8|0)$  and  $E(3|3|10)$ .

- Calculate the angle  $\alpha = \sphericalangle(HGF)$ .
- Calculate the angle  $\beta$  between the edge  $CG$  and the plane  $ABCD$ .
- Calculate the distance from the point  $H$  to the plane  $ABFE$ .





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**Task 15**

4 points

The Niveau Company produces body care products. As a merchandise appeal for their body milk, they include a voucher for a sun cream in every tenth package. These products with vouchers are merged randomly among the other products. Andrea buys one body milk each month. Find the probability that she gets...

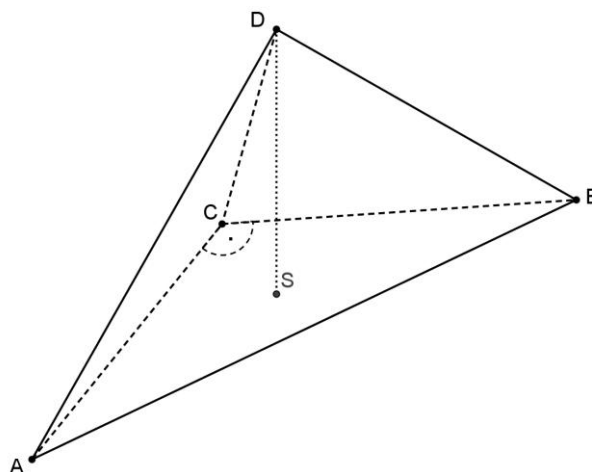
- a) ... the first voucher at the tenth month,
- b) ... the first voucher at the latest at the tenth month,
- c) ... the first voucher at the earliest at the tenth month,
- d) ... exactly two vouchers in ten months.



**Task 16**

6 points

The pyramid  $ABCD$  has a volume of  $V = 12'150$ . The base area  $ABC$  is an isosceles right triangle with a vertex  $A(37|20|16)$  and the apex  $C(0|0|0)$ . The vertex  $B$  lies on the plane  $z = 13$ .



- a) Find the coordinates of  $B$ .  
(State the solution with whole numbers only.)

*If you could not solve a), solve b) using  $B(32|-80|26)$ .*

- b) The perpendicular line dropped from the vertex  $D$  onto the plane  $ABC$  passes the centroid  $S$  of the triangle. Find the coordinates of  $D$  (one solution is enough).



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**Additional space for longer solutions**

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