Matura Exam 2011
Mathematics
Part 1

Class: 4SeWe
Teacher: Ae
Time: 90 min.

Name: _____ points part 1: _____

points part 2: _____

total points: _____ grade: _____

The "*Mathematics and Physics Formulary*" from Compendio is the only permitted aid. The solutions should be presented clearly and well-arranged. Incomplete approaches may lead to a deduction of points.

Exercise 1 3 points

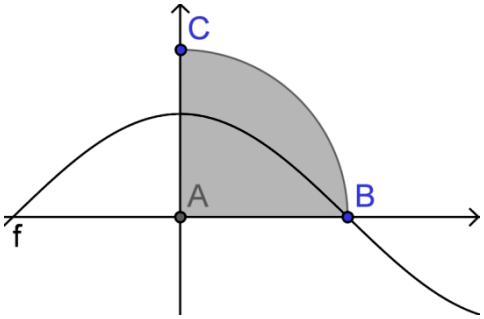
Given the function $f(x)=\frac{x^2-4x+3}{x^2-4x}$, find the equation of the tangent to the graph of this function at x = 1.

Exercise 2 4 points

Given the function $f(x) = a \cdot \cos\left(\frac{1}{2}x\right)$.

a) State the coordinates of B.

b) Find the value of a, if the graph of f cuts the shaded area of the quarter-circle in half.



Exercise 3 5 points

Given the plane P: $2x + y - 2z = 0$, the point $A(6|-2|0)$ and the origin $O(0|0|0)$.

a) Find an equation of the plane which has the same distance from A and O. What is the particular position of this plane?

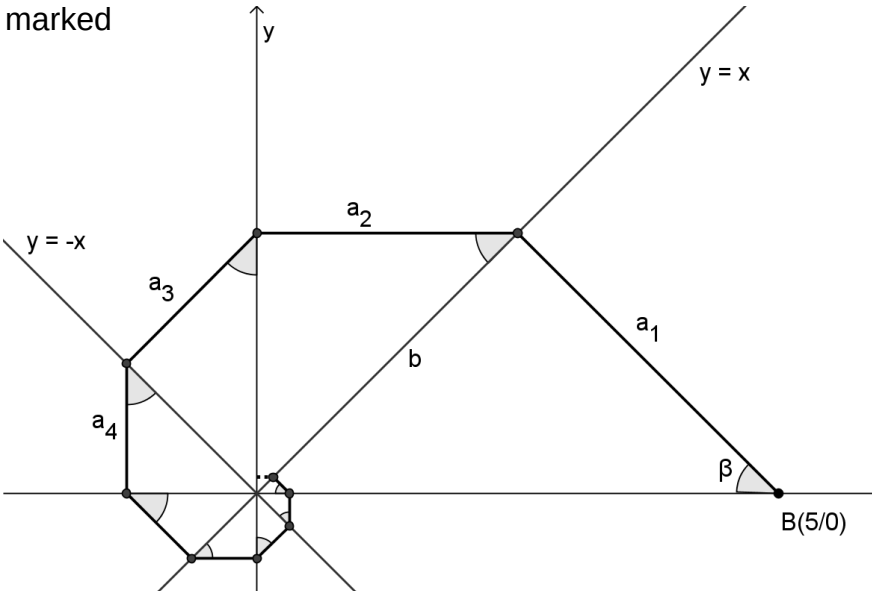
b) Find the equation of a sphere which touches the plane P at the origin and goes through the point A.

Exercise 4 4 points

In the illustration alongside, all marked angles β are equal.

a) Find the length of the infinite spiral for $\beta = 45°$. Simplify your result.

b) For which values of β does the length of the spiral converge?



Exercise 5 3 points

Given the two planes E: $x+2y-2z-3=0$ and F: $4x-3y-5=0$, find the centers of the spheres which touch both planes and have their midpoint on the y-axis.

Exercise 6 3 points

Bart and Milhouse are shooting at a target. Both have one single shot. As Bart shoots better, his probability of hitting is twice Milhouse's probability of hitting. The probability that the target gets hit at least once is $p = \frac{5}{8}$.

Find Bart's probability of hitting.

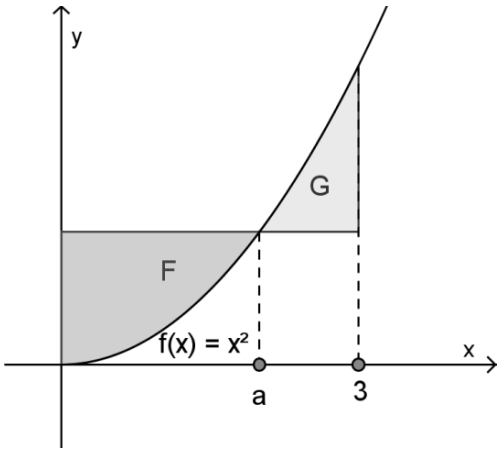
Exercise 7 6 points

Given the function $f(x) = x^2$, where $x \geq 0$,

a) calculate the sum F + G of the areas F and G in the case of a = 2.

b) For which value of a is F = G?

c) For which value of a is F + G minimised?



Matura Exam 2011
Mathematics
Part 2

Class: 4SeWe
Teacher: Ae
Time: 90 min.

Name: _____ points part 2: _____

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Exercise 1 3 points

The points A(1|2|3), B(-16|32|9) and C(-22|22|42) are the vertices of a triangle.
Deduce its shape (general, isosceles, isosceles and right-angled, or equilateral).

Exercise 2 5 points

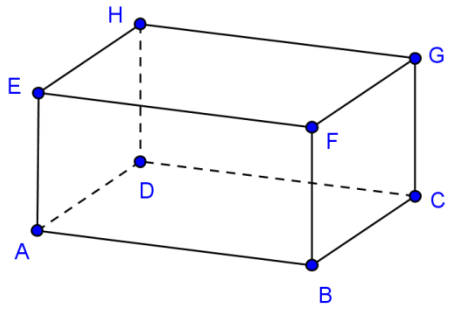
You buy ten “Kinder Surprise” Eggs, which have, in average, one special toy in every seventh egg. What is the probability of winning…

a) … exactly 7 b) … less than 2 c) … at least 1 …. special toys?

d) As you buy more eggs, your probability of “winning at least one special toy” will also increase. What is the minimum number of eggs you would need to buy if you wanted to increase this probability to more than 99%?

Exercise 3 5 points

A cuboid ABCDEFGH has the corners A(3|2|-1) and C(-1|4|5).



The point B lies on the line $l: \vec{r} = \begin{pmatrix} 1 \\ 2 \\ 9 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$

a) Find the coordinates of B.

For the following exercise, use the point B with the smaller z-coordinate.
If you could not solve a), use the wrong point B(-2|1|1).*

b) The corner F lies in the plane P: $4x + 8y + 5z - 38 = 0$. Find its coordinates.

Exercise 4 4 points

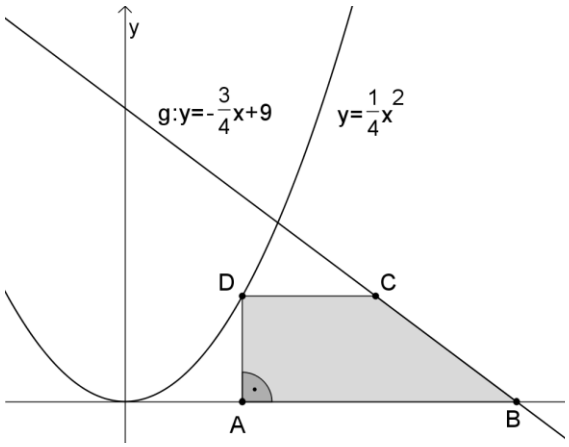
A flea is jumping forwards on a straight line. Its first jump is exactly 1 m long. However, jumping is quite exhausting, and therefore the distances of the jumps decrease.

a) Suppose that the distances decrease with each jump by the same percentage. How much can this percentage decrease be in maximum, if it should reach at least 13 m after 20 jumps?

b) Suppose that the flea decreases its jump length by 1.25 cm with every jump, until it does not jump anymore. What is the total distance it will reach?

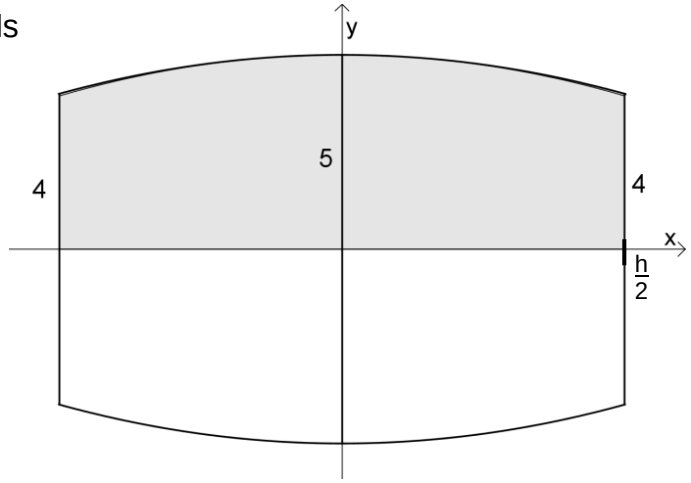
Exercise 5 4 points

Calculate the coordinates of the point A (on the positive x-axis), so that the area of the trapezium ABCD is maximised.



Exercise 6 5 points

A distillery wants to sell whiskey in small barrels which are made of staves (planks) which have the shape of parabolas.
The radius of the cap and the base should be 4 cm, and the radius of the cross section in the middle 5 cm (see the figure).



a) If the height of the barrel is h = 8 cm, what is its volume (in dl)?

b) If the volume should be 1 litre, what would be the height h?