

The "Mathematics and Physics Formulary" from Compendio is the only permitted aid. The solutions should be presented clearly and well-arranged. Incomplete approaches may lead to a deduction of points.

Exercise 1

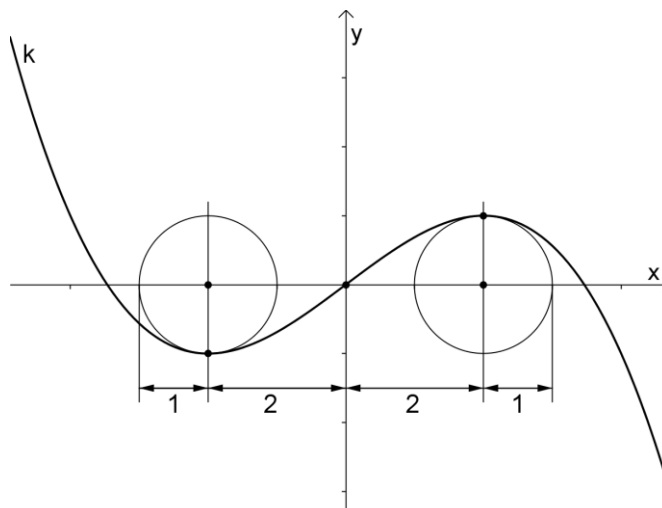
3 points

The sum of an infinite geometric series is 1.5 times the sum of its first two terms. Find the ratio of this geometric series.

Exercise 2

4 points

The curve k in the figure shown is the graph of a polynomial of third order. Find its equation.



Exercise 3

5 points

$C(1|10|0)$ is the apex of an isosceles triangle. Its base AB lies on the line

$$l: \vec{r} = \begin{pmatrix} 0 \\ -6 \\ 0 \end{pmatrix} + t \begin{pmatrix} 2 \\ 6 \\ -3 \end{pmatrix} \text{ and has length } \overline{AB} = 14.$$

- Find the coordinates of the foot of the perpendicular from C to AB .
- Find the coordinates of the points A and B .

Exercise 4

4 points

$A(0|0), B(1|0), C(1|1), D(0|1)$ are vertices of a square. Find the equation of a parabola which passes through A and $E(2|0)$, and which halves the area of this square.

Exercise 5

5 points

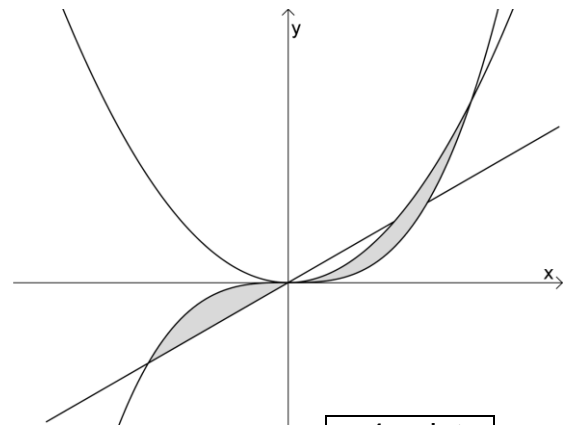
- a) Find the Cartesian equation of the perpendicular bisecting plane P of the line segment \overline{AB} with $A(3|13|8)$ and $B(-4|1|15)$.
- b) Describe the particular position of the plane $Q: 6y - 7z = 0$.
- c) Find the vector equation for the line of intersection s of the planes P and Q .

Exercise 6

4 points

The figure alongside shows the graphs of the functions $f(x) = x^3$, $g(x) = x^2$ and $h(x) = a \cdot x$.

For which value of a are the two shaded areas equal in size?

**Exercise 7**

4 points

Given the function $f(x) = 2 + a \cdot \sin(x)$.

- a) Determine a , so that the tangent at $x = \pi/4$ has a slope of $\sqrt{2}$.
- b) Find the area in the first quadrant enclosed by both coordinate axes and the graph of f .
If you could not solve (a), use $f(x) = 5 + 5 \cdot \sin(x)$.

Exercise 8

5 points

The line through the points $A(-9|7)$ and $B(6|-8)$ intersects the circle $c: x^2 + y^2 - 2x + 8y - 8 = 0$ with centre C at two points P_1 and P_2 .

Find the area of the triangle CP_1P_2 .

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Exercise 1

4 points

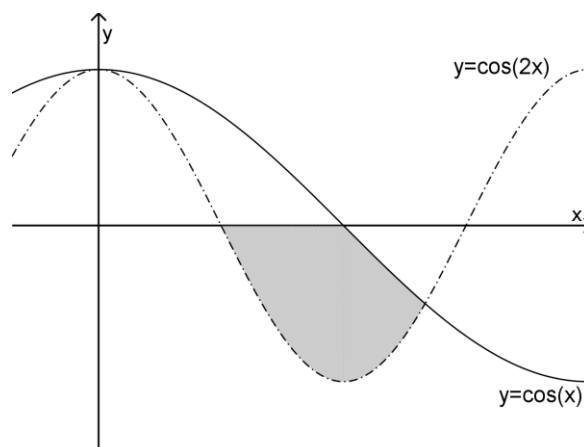
The probability that a newborn is a boy is 0.514.

- What is the probability that two newborn are boys?
- Find the probability that from 60 newborns exactly 24 are girls.
- What is the minimum number of births so that the probability that at least two girls were born is greater than 99%?

Exercise 2

4 points

Find the size of the shaded area.

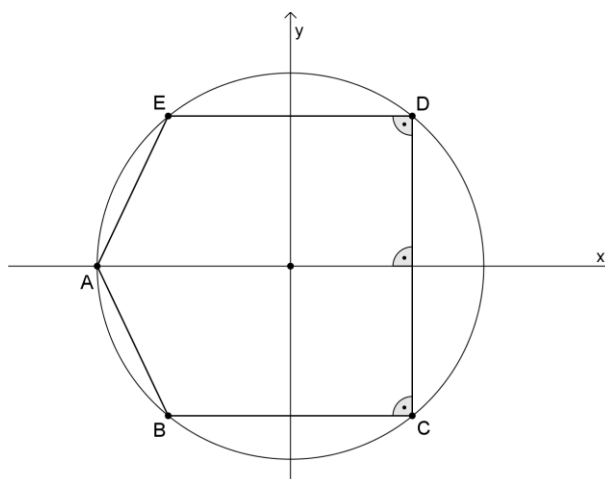


Exercise 3

4 points

A five-sided figure with maximum area is to be inscribed into a circle with radius 1, conforming the figure shown.

Find the x-coordinate of D.



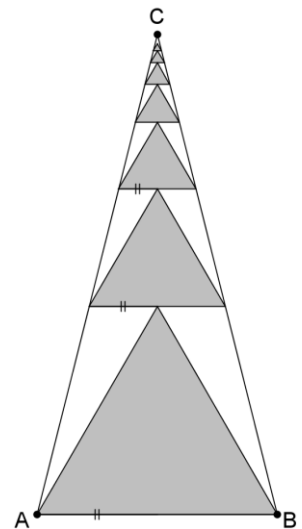
please turn over

Exercise 4

5 points

The given isosceles triangle ABC has a base $\overline{AB} = 14$ and the legs $\overline{AC} = \overline{BC} = 25$. Infinitely many equilateral triangles are inscribed into ABC , conforming the figure shown.

- Find the sum of the perimeters of the first 10 equilateral triangles.
- Find the sum of the areas of all equilateral triangles.

**Exercise 5**

5 points

The points $A(0|1|0)$, $B(1|1|1)$, $C(2|-3|0)$ and D are vertices of a pyramid $ABCD$.

The vertex D lies on the line $l: \vec{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + t \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$.

- Find the area of the triangle ABC .
- For which points D on l is the distance to the plane ABC equal to 1?
- Find the volume of this pyramid.

Exercise 6

3 points

A dice is rolled five times. Find the probability of throwing

- at least one six
- exactly two six
- five different numbers

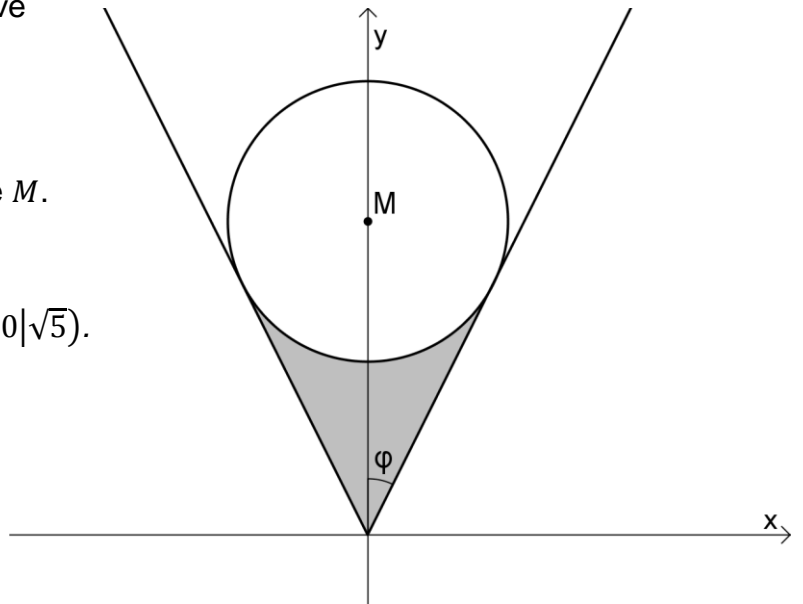
Exercise 7

4 points

The circle with a radius of 1 touches the two lines which pass through the origin and have slopes of ± 2 .

- Calculate the angle φ .
- Find the coordinates of the centre M .
- Find the size of the shaded area.

If you could not solve (b), use $M(0|\sqrt{5})$.

**Exercise 8**

4 points

A car is travelling at night along a street shaped like a parabola with its vertex at the origin (see the figure). The car starts at a point 100 m west and 100 m north of the origin and travels in an easterly direction. There is a statue located 100 m east and 50 m north of the origin.

At what point on the street will the car's headlights illuminate the statue?

