Problem 1: 14 Points

Given is the function f by its function equation $f(x) = -\frac{1}{2}x^3 + \frac{3}{2}x^2$.

a) Calculate the zeroes, the coordinates of the high and the low point as well as the coordinates of the inflection point of the graph of f .

b) The y-axis, the graph of f and the tangent to the graph of f in the high point border a finite area located in the 1st quadrant. Determine the size of this area.

Now the parameter $a \in \mathbb{R}$ is introduced. We consider the family of functions f_a, which is given by the function equation $f_a(x) = -ax^3 + (a+1)x^2$.

c) Prove that the point $P(1 / 1)$ lies on the graph of f_a independent of a.

d) Determine the value of a such that the function f_a has a maximum for $x = 4$.

Problem 2: 15 Points

Given are the points

A(8 / 11 / 11) *C*(-1 / -1 / -1)

P(5 / -1 / -13) *Q*(13 / 7 / -9)

The line *g* runs through the points *A* and *P*.

The line *h* runs through the points *C* und *Q*.

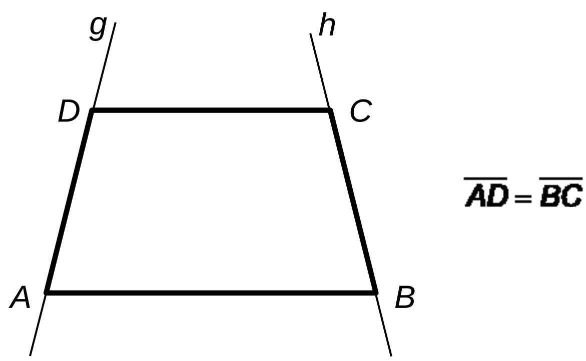
a) Determine the equations of the lines *g* and *h*.

b) Calculate the point of intersection *S* of both lines.

c) At which angle α do the two lines *g* and *h* intersect?

d) The lines *g* and *h* both lie in the plane *E*. Determine a Cartesian equation of this plane *E*.

e) The aforementioned points *A* and *C* are the vertices of the isosceles trapezoid *ABCD*. The points *A* and *D* lie on the line *g*. The points *B* and *C* lie on the line *h*. Calculate the coordinates of the points *B* and *D*.



Problem 3: 12 Points

Given is a wheel of fortune with numbers 1 to 9. The probability to appear is the same for each number.

The wheel of fortune is spun 3 times. Calculate the probabilities for the following outcomes:

a) All 3 numbers are even.

b) All 3 numbers are different.

c) At least one number is bigger than 6.

d) There are exactly 2 different numbers.

Now the wheel of fortune is spun 80 times.

e) What is the probability that the outcome is a square number at least 15 times but not more than 26 times?

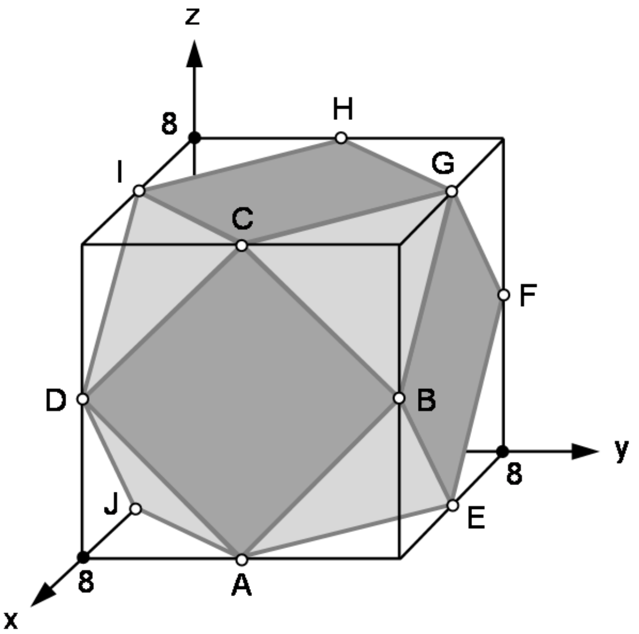
Finally Walter and Gabriel play the game “number cube vs. wheel of fortune“:

Walter pays a wager of $2. Gabriel then spins the wheel of fortune and by doing so determines a number. Walter now rolls a fair six-sided number cube 3 times.
If at least once a number that appears on the number cube equals the one determined by Gabriel, Walter gets his wager returned and additional $4. Otherwise his wager is lost.

f) Which average profit or loss can Walter expect from this game in the long run?

Problem 4: 12 Points

A cuboctahedron is obtained by connecting the centers of the edges of a cube according to the figure shown below.



a) How many faces, edges and vertices does a cuboctahedron have?

Take note that in all following sections the cube as shown above has edges with length 8.

b) Calculate the length of an edge of the cuboctahedron.

c) Determine an equation of the plane E_1 that is given by the vertices *C*, *D* and *I* .

d) Which vertices of the cuboctahedron lie in the plane E_2?

$$E_2: \quad \vec{r} = \begin{pmatrix} 8 \\ 8 \\ 4 \end{pmatrix} + s \cdot \begin{pmatrix} 0 \\ -4 \\ 4 \end{pmatrix} + t \cdot \begin{pmatrix} -4 \\ 0 \\ 4 \end{pmatrix}$$

e) Determine the coordinates of the perpendicular foot of point *P*(14 / 13 / 14) on the plane E_2 .

Problem 5: 8 Points

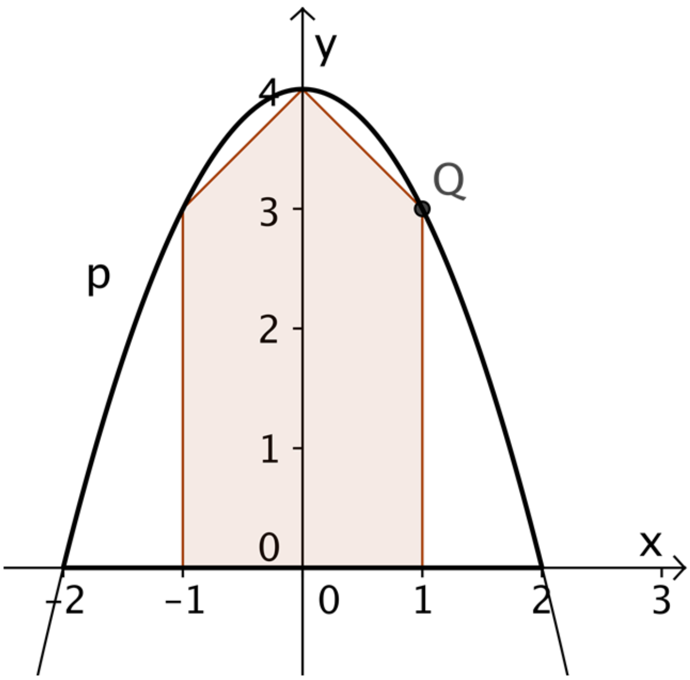
A 'house shaped' area, which is symmetrical about the y-axis, is inscribed into a parabola shaped arch as shown in the figure below.

a) Determine the equation of the parabola p in the figure below!

b) Now one corner of the area is at $Q(1 / 3)$. What is the size of this area?

c) The position of the point Q can be varied along the parabola such that the x-coordinate of Q can assume values between 0 and 2.

How must the x-coordinate of Q be chosen such that the size of the area is maximal?



Problem 6: 14 Points

Given is the function *f* by its function equation $f(x) = 4 \cdot ln(e - x)$.

a) Calculate the intersections of the graph of *f* with both coordinate axes. Indicate the coordinates exactly.

b) Determine the equation of the normal line to the graph of *f* at the intersection with the *y*-axis.

At which angle do the normal line and the y-axis intersect?

c) Confirm by a calculation that the function given by $g(x) = e - e^{0.25x}$ is the inverse function of *f* .

d) The graph of *f* and both coordinate axes border a finite area in the first quadrant. When is area is rotated about the ***y*-axis** a solid is formed that resembles the bowl of a champagne flute which is turned up-side down.
Calculate the volume of this solid.

Hint: Before you start calculating the integral of revolution you should expand the function term!